TITLE
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Correlations in a random universe

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Abstract. The Large Scale Structure battle as seen by one of the footfolk, followed by some remarks on things vaguely seen through the gunsmoke.

1. After the battle

When the battle is over, the generals begin to write their memoirs. These memoirs are full of lies, because the generals – who are almost always among those who survive the war – are worried about their place in history.

When a soldier writes memoirs, these are not very reliable either, because a soldier sees only a small part of the field, has a shortage of everything, and is mostly scared silly.

I am a soldier who served under the great Jan Oort, whose hundredth birthday we commemorated last April. I have seen only a small part of the cosmology battle and anyway I am mostly campaigning in a different corner of the field now. So you can imagine my pleasant surprise when Vicent Martínez invited me to speak here. I am going to talk about three things: first, my past involvement with the theory of large scale structure; second, my present involvement with this; and, third, some remarks about what I think is the biggest enigma today, as before: the cosmological constant.

2. Between Varenna and Leiden

Just as Bernard Jones did yesterday, let me say a few words about the way in which I got involved in cosmology. I studied theoretical physics in Utrecht under Tini Veltman, who won the Nobel Prize in physics last year, together with Gerard 't Hooft. Because job prospects in that field were very bad, I also did a degree in astronomy. I graduated in 1969; Oort invited me to Leiden. In those days, as opposed to the strictly regimented pseudo-industry that astronomy has become today, picking a thesis subject was up to the individual. Oort's colleague Van de Hulst told the story that he once spoke with the physicist Gorter, and worried about choosing a subject. Whereupon Gorter said: "Don't worry, the subject will pick you."

And so it was. Already in Utrecht I had become interested in the formation of structure in the Universe, from stars to galaxies, and my participation in the Varenna Summer School (1968; Sachs, 1971) which Bernard mentioned was extremely important. There I met not only Bernard Jones, but also Kip Thorne, Virginia Trimble, George Ellis, Joe Weber, and Jürgen Ehlers. To give you an idea of the atmosphere: at the end of Ehlers's lecture series, the students gave him a standing ovation, and we did not stop clapping until he had left the chapel where the lectures were being given. But the most important person there (for me) was Martin Rees, after Oort the most influential person in my scientific life.

3. Leiden Style

When I got to Leiden, Oort gave me a paper to read that he had written on the origin of the rotation of galaxies. A week later he asked me and another graduate student – whose name I will keep secret – into his office, and asked for our comments. I told him that I thought the paper was no good, because it violated the laws of hydrodynamics. The other guy seemed to be frozen solid, but I was too ignorant to be afraid. That sometimes happens to soldiers. Moreover, being Veltman's student, I had acquired a fearlessness that was quite real, even though it was not warranted by my stature. Oort looked at me across his large desk, and asked "Well, do you know how to do it, then?" And I said "Yes, professor." Whereupon he stood up, fixed me from under his bushy eyebrows, and said "Then I will be looking forward to seeing your results."

4. The Oort Constants of the Universe

The reason that I was so over-confident was, that I had learned hydrodynamics during my stay at Utrecht, under the guidance of Van Bueren and Kuperus. I had become totally fascinated with this trade, wrote a master's thesis on nonlinear magnetohydrodynamic wave coupling, and was eager to apply this wondrous toy on the largest scales I could imagine. I was interested in big things, because it was clear that the formation of individual galaxies and other small stuff was likely to be dominated by radiative phenomena, and that the clean interplay between gravity and hydrodynamical effects, such as pressure and dissipation, would only be found beyond megaparsec scales.

That is true, even to this day. The giant simulations about which we hear so much, almost all use the technique of smoothed-particle hydrodynamics (SPH), which is a very superficial approximation of hydrodynamics. Leon Lucy invented it for the specific purpose of (binary) star formation, and for this it was an appropriate and indeed clever approximation; but its excessively dissipative behaviour is, and always will be, utterly unsuitable for resolving true hydrodynamical details such as shocks and other discontinuities. And the cosmic SPH models contain no radiation at all; so I think that these monster computations do not deserve to be taken nearly as seriously as most people take them.

So, more than in individual galaxies – which was Oort's approach – I was interested in really big stuff. I knew that the shape of self-gravitating things would be highly asymmetrical, because of the following effect. Imagine that we simplify the formation of galaxies in an infinite Universe to the formation of just two objects of equal mass, forming out of a spherical volume. Cut a sphere with radius R out of an infinite self-gravitating Universe, and pack the matter from that sphere into two equal point masses $M/2$, placing them symmetrically on opposite sides of the centre of the evacuated sphere, a distance r from the centre. What is the net force on the masses? The points attract each other with a force

$$F_{\rm in} = -\frac{GM^2}{4(2r)^2} \tag{1}$$

The void acts as a negative mass with constant density, so that each mass experiences a force

$$F_{\rm out} = \begin{cases} \frac{GM^2}{2R^3}\, r & \text{inside the cavity} \\ \frac{GM^2}{2r^2} & \text{outside the cavity} \end{cases} \tag{2}$$

(actually, in Newtonian gravity this pseudo-repulsion is due to a surface term in the potential, which is the first cousin of the cosmological constant, but let that pass). The net force is zero if

$$r = \frac{1}{2}R \tag{3}$$

If the condensations are closer together than this, they will start to fall towards each other; if they are farther apart, their mutual attraction is not strong enough to overcome the effects of the 'sea' of matter surrounding them. Consequently, I expected that the pieces of a perturbation that are close together will collapse onto each other faster than the faraway parts, and that thereby the collapsing shape would become more and more irregular. On the other hand, an irregular void (from which the condensing matter came) would have to become more and more spherical in the course of time.

Oort had given me a picture of the Shapley-Ames survey of galaxies, dating from 1933 (remember, this was in 1969!) and had pointed out the elongated shape of the arrangement of galaxies in the direction of the Virgo cluster. This seemed to be in accord with the above argument, but more detailed analysis was necessary. It seemed natural to study the formation of asymmetric objects by means of the deformation tensor, i.e. the kinematic field induced by a velocity field v_i:

$$\mathcal{Q}_{ij} \equiv \frac{\partial v_i}{\partial x^j} \tag{4}$$

This was all the more natural in the Leiden environment because Oort himself had used this approach when studying the motion of stars; in cylindrical coordinates, the deformation quantities reduce to the 'Oort Constants'. The tensor $\mathcal{Q}$ can be decomposed into irreducible parts, yielding a 'generalized Hubble law':

$$D \equiv \sum_i \mathcal{Q}_{ii} \quad \text{(diagonal, expansion)} \tag{5}$$

$$R_{ij} \equiv \tfrac{1}{2}(\mathcal{Q}_{ij} - \mathcal{Q}_{ji}) \quad \text{(antisymmetric, rotation)} \tag{6}$$

$$S_{ij} \equiv \tfrac{1}{2}(\mathcal{Q}_{ij} + \mathcal{Q}_{ji}) \quad \text{(symmetric, shear)} \tag{7}$$

In the case of the Oort Constants, the divergence D is the 'K term', which is negligible because the Galaxy doesn't expand; the other two are the famous A and B constants. In the Universe, the 'K term' is the Hubble expansion, which totally dominates the other two. But on the basis of the argument leading to Eq.(3) I expected that in a fragmenting universe, the 'Oort Constants of the

Universe', R and S, would become very large. Oort, in turn, was intrigued by R, because of his interest in the origin of the rotation of galaxies, with which (as I indicated above) our acquaintance began.

5. Ellipsoids

At the time, I didn't know any better than to compute numerically the gravitational collapse of homogeneous ellipsoids. The results confirmed my supposition that the primary collapsing structures would be highly asymmetrical. By now, we have reached 1970, and you may wonder why I do not mention the superb Russian work on this subject. The fact of the matter is, that I was ignorant of it. I know that this is no excuse – stupidity never is. Only later did I meet the great Yakov Barisovich Zel'dovich, and some of his great students, such as Igor Dimitriyevich Novikov. At the time, I wasn't yet aware of the work by Zel'dovich and his co-workers, an omission which I came to regret.

However, I had found a nice paper by Lin, Mestel & Shu (1965), who had discovered a much more general version of the argument leading to Eq.(3). The potential Φ near any point (x, y, z) of a self-gravitating medium can be written as

$$\Phi = \sum_{ijk} a_{ijk} x^i y^j z^k \tag{8}$$

Near a density maximum, the leading terms are the quadratic ones, which, by a suitable orientation of Cartesian coordinates, can be written as

$$\Phi = Ax^2 + By^2 + Cz^2 + \cdots \tag{9}$$

Neglecting terms of higher than second order, this is the potential of a homogeneous ellipsoid. That should be no surprise: the smallest closed contours in any topographical map are ellipses.

The collapse of high-density regions can thus be approximated by considering the homologous motion of ellipsoids. Consider, then, the Newtonian collapse of a homogeneous ellipsoid. Suppose that a particle within such a mass distribution were initially located at (a, b, c), and that at some later time t it had moved to the point $(aX(t), bY(t), cZ(t))$, then the density ρ would evolve according to

$$\rho(t) = \rho_0 / XYZ \tag{10}$$

The equations of motion for the scaling functions X, Y, and Z are found as follows. The potential Φ obeys

$$\Phi = k(\alpha x^2 + \beta y^2 + \gamma z^2) + \mathcal{O}(3) \simeq k(\alpha a^2 X^2 + \beta b^2 Y^2 + \gamma c^2 Z^2) \tag{11}$$

and Poisson's Equation demands that

$$k(\alpha + \beta + \gamma) = 2\pi G \rho \tag{12}$$

The components of the gravitational force are $-\partial\Phi/\partial x = -\frac{1}{X}\partial\Phi/\partial a$ *et cycl.*, so that the equations of motion become

$$-\frac{1}{X}\frac{d^2X}{dt^2} = 2\pi G\rho\alpha \ ; \quad -\frac{1}{Y}\frac{d^2Y}{dt^2} = 2\pi G\rho\beta \ ; \quad -\frac{1}{Z}\frac{d^2Z}{dt^2} = 2\pi G\rho\gamma \tag{13}$$

where the functions α, β, and γ are defined as

$$\alpha = abc \int_0^\infty \frac{ds}{(a^2+s)\Delta} \quad \text{et cycl.} \tag{14}$$

in which

$$\Delta^2 \equiv (a^2+s)(b^2+s)(c^2+s) \tag{15}$$

Here a, b, and c are identified with the axes of the ellipsoid.

Now comes a crucial observation: without loss of generality, one may order the axes according to $a > b > c$, in which case $\alpha < \beta < \gamma$, so that

$$-\frac{1}{X}\frac{d^2X}{dt^2} < -\frac{1}{Y}\frac{d^2Y}{dt^2} < -\frac{1}{Z}\frac{d^2Z}{dt^2} \tag{16}$$

Consequently, the axial ratios $a : b : c$ always increase with time, and *slight initial asphericities are amplified during the collapse.* This secular increase of aspherical perturbations provides an explanation for the filamentary appearance of megaparsec structures. Although the authors did not look beyond galactic scales, their theorem was eminently suited for megaparsec structures. For the contraction described, the velocities inside the ellipsoid are linear functions of position: *the collapse produces a Hubble-type velocity field.* Moreover, Poisson's Equation (12) implies that the potential function is always smoother than the corresponding density distribution (in Fourier terms: the difference is two factors of the wave number), so that the supposition in Eq.(9) is even better than one might think.

I tried to find out if the collapse of elongated structures could be observed in the Virgo cluster and the Perseus cluster. Oort was extremely skeptical of this. During my thesis defense, he asked, in the formal manner which is customary during these full-dress Dutch occasions: "Mr. Candidate, you really maintain that there are structures as big as 14 megaparsec?" To which I could only respond, in similarly formal fashion: "Most learned opponent, indeed I do." Oort shook his head. He did not believe in such gigantic things. Nor did he believe in Zwicky's dark matter, which provided an indication of enormous amounts of stuff lurking under the visible surface. In a way this was strange, because Oort himself had found evidence for dark matter in our Galaxy. I did accept its existence; perhaps this was because of Zwicky's angry no-bullshit style, which I secretly admired. Remember that Zwicky was a guy who had built his own Schmidt telescope, long before other people inderstood the importance of that optical design.

Oort thought about galaxy clusters as he did about star clusters such as the Hyades, on which Van Bueren did remarkable work under Oort's guidance. After a bit of to-and-fro, he allowed me to publish the Virgo cluster stuff (Icke 1973), but he forbade publication of the Perseus cluster material (Fig.1). "That is not up to the Leiden standard," he said. My difficulty was, that in those regions where you can see the galaxies easily, the space density is high; therefore, the velocity dispersion is high, and this makes it very difficult to observe the R and S terms in Eqs.(6-7). You may imagine my chagrin when, many years later, it turned out (in the work of Giovanelli & Haynes, 1985, 1991) that the Perseus region contains a chain of galaxies even more spectacular than Virgo.

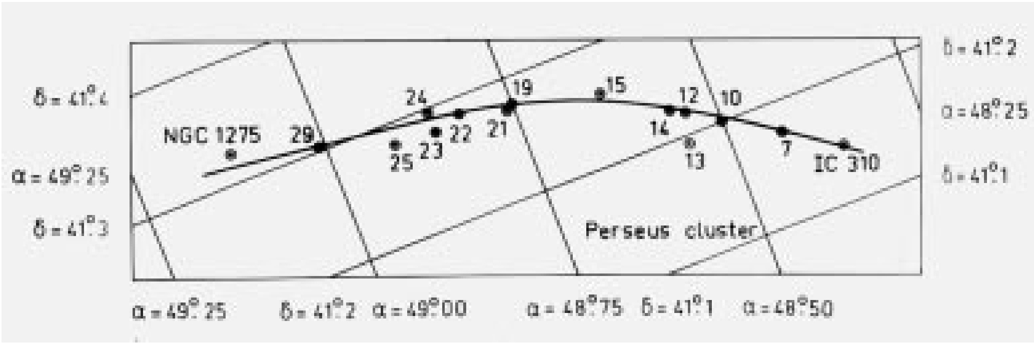

Figure 1. The Forbidden Diagram: my 1970 interpretation of the Perseus Cluster which was panned by Oort. This image was scanned from a lantern slide; the original, which I drew by hand (no plotting programs, and no draughtsmen working for grad students!) has been lost.

6. The Voronoi Surprise

So much for my involvement with the early history of dark matter, which perhaps I ought to call the dark history of early matter. I will return to this dark history at the end of my talk.

Throughout these events, it had been obvious to me (because of Eq.(3)) that there should be enormous empty regions in between the galaxy filaments. After all, the Oort constants of the Universe showed that there must be zones of super-Hubble expansion; and the galactic matter must have come from somewhere.

It was not until 1983, though, that I finally wrote this up (Icke 1984). During an AAS meeting in that year, I was talking with Bernard Jones, Joan Centrella, and Adrian Melott. C & M had just completed (1983) the first three-dimensional hydrodynamic simulation of dark matter-dominated structure formation. They were puzzled that the low density regions were so spherical, whereas the collapsing structures were long filaments. Of course they knew that Zel'dovich had emphasized the formation of 'pancakes' (*blini*), but these turned out to be hard to find in their results.

And so, during coffee break, and in the same brazen manner with which I had talked to Oort a decade before, I said that it was obvious: in order to avoid nonlinearities and other complications that always occur in high-density regions, we should view the development of structure in a selfgravitating pressure-free medium by considering the evolution of the *low*-density regions. These are the progenitors of the observed voids. The arguments presented by Lin *et al.* can still be applied, except that the sense of the final effect is reversed: the $<$ in Eq.(16) must be replace by $>$ if a, b, c are the axes of a void.

Because a void is effectively a region of negative density in a uniform background, the voids expand slightly faster than the average Universe, as a 'superHubble bubble' while the overdense regions collapse. In the process, *slight asphericities of the voids decrease as the voids become larger.* This I called the 'Bubble Theorem', which explained the numerical findings of Centrella & Melott, and Joan and Bernard persuaded me to write it up (Icke 1984). The proof holds strictly only on a non-expanding background, though this should be no objection for structures which are much smaller than the region where the collapse speed is approximately equal to the Hubble speed across the void. Because $|\delta\rho/\rho|$ does not exceed unity in a void, the approximation will remain good for a longer period, except, of course, near the outer parts of the voids, where the matter gets swept up.

According to the above, voids are the dominant dynamical component of the inhomogeneous Universe. One may think of the megaparsec structure as a close packing of spheres of different sizes, out of which matter flows in a slightly super-Hubble expansion towards the interstices of the spheres. Thus, *the importance of the Bubble Theorem is that it provides a specific physical mechanism for producing the non-Poissonian matter distribution in the large scale Universe.* Moreover, it dictates a specific spatial arrangement, and predicts that matter should collect only in the regions where the filaments meet.

Back in Leiden, I managed to get a brilliant young graduate student, Rien van de Weygaert, interested in this problem. Since Rien was graduating in mathematics as well as astronomy, he was very well placed for the next step. I asked him to find out what such a collection of superHubble bubbles would look like. In no time at all, he unearthed a paper almost a century old (Voronoi 1908) that provided the answer: a collection of expanding spheres produces a unique partitioning of space, called a *Voronoi tessellation.* Rien proceeded to give an extremely complete treatment of this, including some remarkable 'experimental mathematics' (Van de Weygaert & Icke, 1989).

Together, we studied the properties of this type of tessellation. We called it 'Voronoi foam', which was perhaps an unfortunate choice because it led some people to think that the walls of this tessellation join at angles of 120°, which is not the case at all. A Voronoi tessellation comes about as follows: given a set of points in space, called *nuclei.* A Voronoi cell around a given nucleus is the set of all points that lie closer to that nucleus than to any other.

Part of the fun is, that this requires a recipe for the distance d. Figure 2 shows a tessellation for the 'Pythagoras recipe'

$$d^2 = x^2 + y^2 \tag{17}$$

A different recipe might be

$$d = |x| + |y| \tag{18}$$

which I call the 'Manhattan distance': it is the distance produced by an infinitely fine mesh of perpendicular streets-and-avenues. This recipe produces a completely different tessellation (Fig.3), in which the cell walls occur only at integral multiples of half a right angle.

There is no reason to suppose that space is anisotropic, so Eq.(17) applies (in three dimensions, of course). The really special thing about the Voronoi model is, that it demands a specific topological arrangement of the matter distribution. There can be *four and only four elements*: voids (the superHubble bubbles), walls (*blini*, pancakes), filaments, and nodes.

When I calculated the two-point correlation function of the 'galaxies' in a Voronoi model in which points were allowed to migrate from voids, via walls and filaments, to nodes, I found that the result looked remarkably like the results found from observed galaxy counts by Peebles (1980) and others. Rien

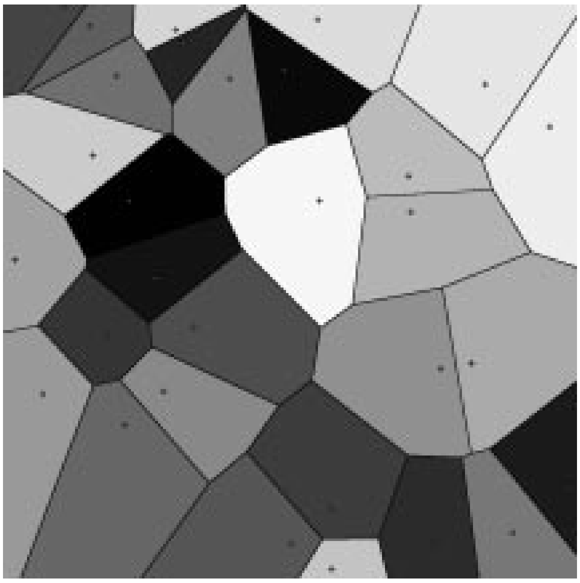

Figure 2. Two-dimensional Voronoi tessellation using the Pythagoras distance recipe (17). The crosses indicate the cell centres. Each Voronoi cell is the locus of all points which are closer to its centre than to any other centre, where 'closer' is measured using some disctance prescription (in this case (17)).

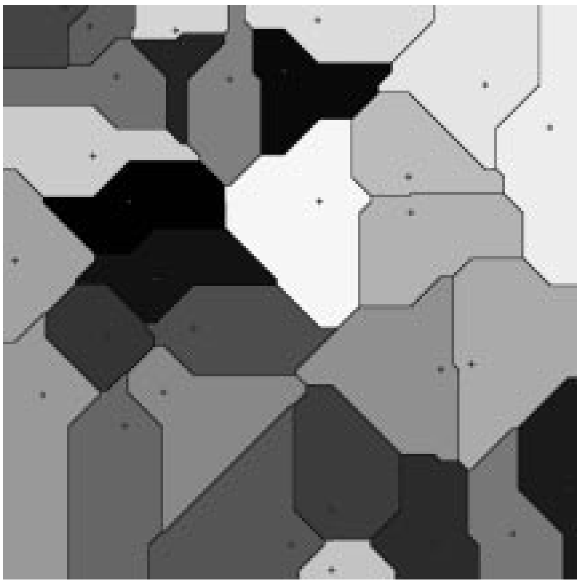

Figure 3. Two-dimensional Voronoi tessellation using the 'Manhattan' distance recipe (18). Notice that this tessellation is not isotropic: due to the absolute-value operations in (18), the preferential directions of the coordinate system are in evidence. The cell walls are oriented at angles which are multiples of 45 degrees; this may be useful in practical applications.

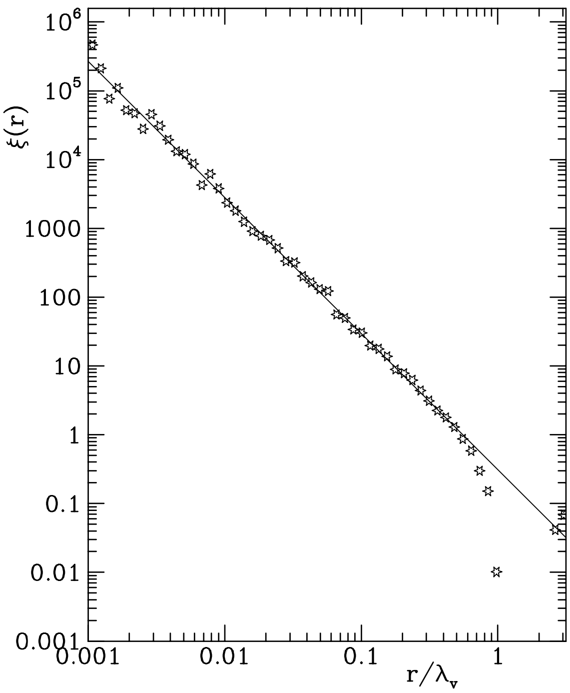

Figure 4. Two-point correlation of Voronoi vertices using the Pythagoras distance recipe. From Van de Weygaert, 2000 (priv.comm.)

proceeded to compute the two-point correlation function of the Voronoi nodes (Fig.4), which we supposed to correspond to the galaxy clusters. This showed two remarkable things: first, that the amplitude of this correlation was larger than that of the galaxies, as had indeed been seen in galaxy surveys but was not understood at the time; second, that this function was a power law over a large range of its argument, with an exponent of the order of -2, close to the value of -1.88 that was observed. So we had found *a way to make correlations from a totally uncorrelated distribution of nuclei* (centres of initial low-density regions).

Furthermore, the rise and fall of the Voronoi features can be calculated exactly. The effective excess Hubble parameter H^* in a Voronoi structure is *the same* in all topological features. This is one of the cute properties of the Voronoi model.

Let N be a nucleus, let P be the point where a *Delaunay line* (that is a line connecting two nuclei, perpendicular to a wall) intersects a wall, let W be a point in the wall, and let the angle WNP be called ϕ (Fig.5). If we indicate the distance NP by a, the distance PW by b, and NW by c, then the velocity at W is cH^*. The components perpendicular and parallel to the wall are

$$cH^* \cos\phi = aH^*; \qquad cH^* \sin\phi = bH^* \tag{19}$$

Thus, *the excess velocity in any Voronoi feature is simply found by multiplying H^* with the length along the feature.* This allows us to use the above formula for $N = 3$, 2, 1, and 0.

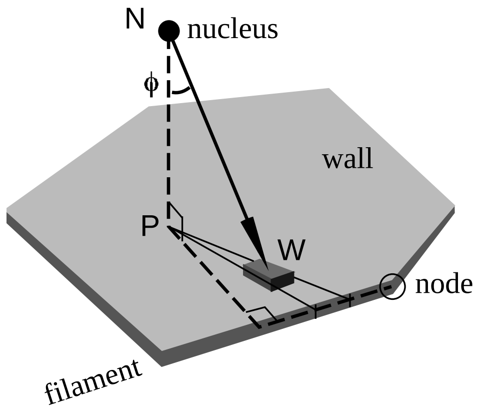

Figure 5. Geometry of the mass flow in a three-dimensional Voronoi tessellation.

It is then easy to determine how matter is transferred from one Voronoi feature to another. In the quasi-linear case, the Hubble parameter H^* is proportional to $t^{-1/3}$, because the excess velocity increases as $H^*ax = v \propto t^{1/3}$, and in the Einstein-De Sitter case the Universe's scale factor $a \propto t^{2/3}$. The power law dependence means that one may absorb H^* into the time by defining $d\theta \equiv H^* dt$. Consequently,

$$\theta = \theta_r (t/t_r)^{2/3} \tag{20}$$

where t_r is the time at which the Universe becomes transparent to radiation and may begin to fragment. The constant θ_r is related to the amplitude δ_r of the voids at decoupling; one readily finds that

$$3\theta_r = \delta_r \tag{21}$$

The amount dm of mass lost in a dimensionless time interval $d\theta$, in N dimensions, is then

$$dm = -Nm\, d\theta \tag{22}$$

The mass gain is found simply by reversing the sign of the loss term of the feature higher in the hierarchy. This gives the following equations for the mass in voids, walls, filaments and nodes:

$$\frac{dm_v}{d\theta} = -3m_v \tag{23}$$

$$\frac{dm_w}{d\theta} = 3m_v - 2m_w \tag{24}$$

$$\frac{dm_f}{d\theta} = 2m_w - m_f \tag{25}$$

$$\frac{dm_n}{d\theta} = m_f \tag{26}$$

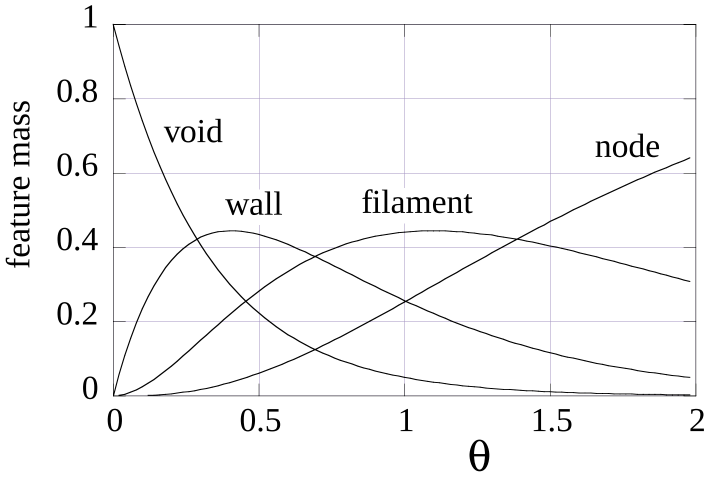

Figure 6. Growth and decline of the mass in the four types of Voronoi features. The voids steadily (and quickly) lose mass, which ultimately ends up in the nodes. Notice that, if we could observe the mass in the various cosmological mass concentrations, we could determine the age of the Universe by means of this graph.

These equations are simply solved by noting that the N-dimensional mass loss equation has a solution of the type $\psi \exp(-N\theta)$:

$$m_v = e^{-3\theta} \tag{27}$$

$$m_w = 3e^{-2\theta}(1 - e^{-\theta}) \tag{28}$$

$$m_f = 3e^{-\theta}(1 - e^{-\theta})^2 \tag{29}$$

$$m_n = (1 - e^{-\theta})^3 \tag{30}$$

If we call $m_h = m_w + m_f + m_n$ the total mass of the high density regions, while $m_l = m_v + m_w + m_f$ is the mass in low density features, I find that

$$m_h/m_v = \exp(3\theta^{2/3}) - 1 \tag{31}$$

$$m_l/m_n = \left(1 - \exp(-\theta^{2/3})\right)^{-3} - 1 \tag{32}$$

The time scale of the Voronoi growth can be related to the initial amplitude of the perturbations. Both mass ratios can then be compared with published galaxy surveys. This gives us a way to determine 'what time it is' in the simulations and in the Universe (Fig.6). The equations allow one to relate the dimensionless time parameter θ to the cosmic time and to the redshift z at decoupling:

$$\theta = \frac{1}{3}\delta_r (t_0/t_r)^{2/3} (t/t_0)^{2/3} = \frac{1+z}{3}\,\delta_r (t/t_0)^{2/3} = \frac{1+z}{3}\,\delta_r \tau^{2/3} \tag{33}$$

The scaling of the time is given by the initial amplitude.

Van de Weygaert and I have continued working on Voronoi models, revisiting this mathematical beauty spot every now and then for refreshment. During

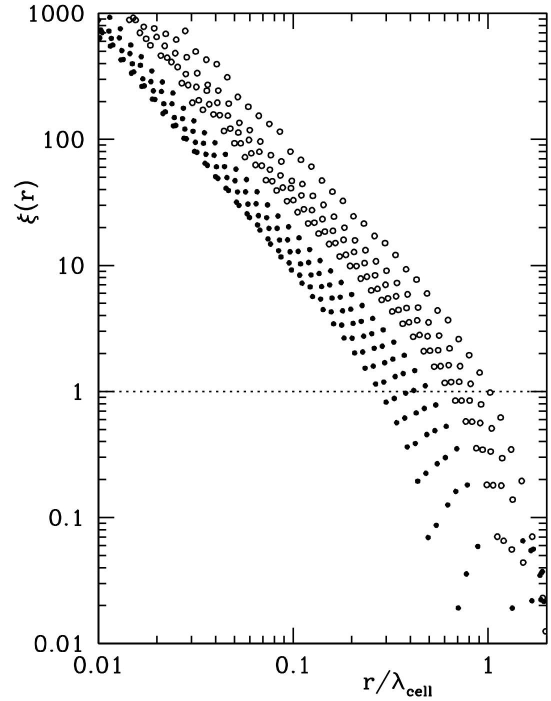

Figure 7. Two-point correlation $\xi(r)$ of the nodes of three-dimensional Voronoi tessellations, by Van de Weygaert (2000, priv.comm.) The lower sequence shows the ξ of the nodes ('Abell clusters'); the higher sequences show the ξ of those that are found in higher and higher density peaks, in steps of one-half sigma. Two things are clear: (1) the clustering amplitude steadily increases as higher and higher density peaks are selected, (2) the shape of ξ tends to a universal form, with logarithmic slope $\gamma = -1.85$.

one of these visits, Rien has discovered another remarkable property of the correlation of Voronoi vertices that I wish to preview here. As I mentioned above, the two-point correlation function of these vertices is rather close to what is observed for clusters of galaxies. In order to see what the properties of the most prominent mass concentrations would be, Van de Weygaert used the vertex distribution as a point process, from which he generated subsets by selecting the high-density peaks. By picking out the peaks above $0.5\sigma, 1\sigma, 1.5\sigma$, and so forth, he obtains a set of point processes, the correlations of which can be studied as before (Fig.7). As expected, the amplitude of the correlation function increases, but the remarkable thing is that *the shape of the function converges to a fixed form.* The exponent of the corresponding power law tends to a fixed value: $\gamma = -1.85 \pm 0.05$, surprisingly close to the observed value.

Although the two-point correlation does not prove this completely, the strong suspicion is that *the distribution of the high-density peaks in a field of Voronoi vertices is self-similar.* Van de Weygaert is now studying the higher-order correlation properties of these point processes to see if this hypothesis still holds.

7. Modern Times

As Bernard Jones explained yesterday, the bad old days were remarkably bad in some respects. Computers were rather primitive then. Today, I could reprogram the chip in the GSM telephone in your pocket, and use it to solve equations of motion fifty times faster than I did when I computed ellipsoid collapse for my Ph.D. thesis.

After 12 years abroad I returned to Leiden and my life changed. Oort had retired 14 years before that time, and my duties became different. He asked me to survey the literature for him, and once a week he invited me into his office for a discussion. You may gather that I still felt rather intimidated by this, perhaps even more so than when I was a youngster who hadn't yet collided with his own limitations. Still, these were immensely valuable years, because I was allowed to continue working in the personal, intimate style that always suited me best. Until his death, the regular conversations with Oort sheltered me from the sea change that was sweeping the field. For in the meantime, the research landscape had changed beyond recognition. Not because of the questions we ask, not even because of the wonderful answers we are beginning to get, but due to the politics of science. What Bernard told you yesterday is entirely correct: present-day research groups are too big, the money they use is too big, and they are run by people who believe of themselves that they are correspondingly big.

I do not necessarily refer to observational astronomy. The enterprise there is always big, it cannot be otherwise. Measured in today's euros, we find a canonical scale of expense: a hundred million is so little that it does not buy you much; ten billion is too much for political reasons; so the state-of-the-art money scale is always of the order of one billion (year 2000) euros. But also the part of the field that used to be called astrophysics has become big; what is worse, it has become big business. And I do not like big business. I happen to believe that all truly important things came not from billion-euro businesses, but from the few pounds of grey glop between the ears of individual people. It is this feeling that was reinforced by Oort's attitude, even though the Old Oak of Astronomy (Chandrasekhar's words in his condolence telegram) had managed to get quite a few acorns out of the Netherlands Treasury.

So I have opted out. In karate, my favourite sport, if you reach a certain stage, you abandon your black belt and start wearing a white belt again. So, from now on, no more talk about the past: let's face the future.

8. The Cosmological Constant

In my opinion, the most annoying – and the most important – question is: is there a cosmological constant Λ, and if so, why, and why a particular value?

Compare this with classical mechanics. There, a gravitational potential Ψ is not observable; only the gradient $\partial\Psi/\partial x$ has consequences. Similarly in electrodynamics: the vector potential cannot be seen, it is the field tensor that is observable. Just so in QED; the photon field itself is not seen; its direct consequences can be removed by the process of renormalization.

But the Einstein equations are different. In those, the analogue of the potential is the metric tensor $g_{\mu\nu}$, and it appears explicitly. This is tantamount to a potential that can be 'seen' directly. In a static universe, it can be renormalized away; in a dynamic universe this can only be done at one specific time. So Λ has dynamical consequences, and because it is a proportionality constant to $g_{\mu\nu}$ it represents a vacuum energy density. I believe that this is a peculiar remnant of the way in which Einstein constructed his equation, namely by linking g_{00} to Ψ and T_{00} to ρ in Poisson's Equation. The bizarre thing is, that Poisson's Equation is a prescription for the *potential*, whereas Einstein's Equation is taken as an *equation of motion* because the matter in $T_{\mu\nu}$ couples to $g_{\mu\nu}$ *and its derivatives.* This is surely odd; in classical as well as quantum mechanics, the equation of motion is connected to the potentials in such a way that only potential derivatives occur, or (in the QED case) only the effects of virtual particles (with a vertex at the both ends of the particle). This removes the absolute value of the potential, or allows renormalization of the vacuum energy density terms.

Current fashion says that Λ has been observed. But when we consider its value, something remarkable appears. In a phase plot of the universal expansion, in which the time derivative da/dt of the cosmic scale factor is plotted against a, we happen to live near the time when the curve reaches a minimum. That is to say, 'round about this time the deceleration of the universal expansion is just about being replaced by the acceleration due to the cosmological constant. It is a crazy miracle that, at the time when we naked apes come climbing out of the trees, these two effects should be nearly equally strong. I do not accept 'anthropic' arguments for anything, and certainly not for this. We cannot compute the structure and the behaviour of even the simplest virus from first principles; so why should I take arguments seriously that hinge on the presence of humans?

To me, Λ is a road sign, but I have no idea what is written on it, or even in which direction it points. But I could make some wild guesses. These are desperate times, and they call for desperate measures.

First, let's say $\Lambda = 0$ after all. Astronomically this is hard. As Zel'dovich would say, in his inimitable voice: "Hit is poossible, but hit is difficoolt." Moreover, since Λ cannot be renormalized away (except in unbroken supersymmetry), we can guess its value. From dimensionality we get

$$[\Lambda] = [\mathrm{sec}]^{-2} \tag{34}$$

Thus, dimensionally, Λ scales as the square of the Hubble parameter H. If m is a mass scale (e.g. from QCD or whatever), then a corresponding length scale is the Compton length

$$\lambda_C = \frac{\hbar}{mc} \tag{35}$$

The corresponding mass density is

$$\rho = \frac{m}{\lambda^3} \tag{36}$$

and therefore

$$\Lambda = G\rho = G\,\frac{m}{\lambda^3} = m^4 c^3 G \hbar^{-3} \tag{37}$$

Employing the Planck units

$$m_P \equiv \sqrt{\frac{\hbar c}{G}}\ ; \qquad t_P \equiv \sqrt{\frac{\hbar G}{c^5}} \tag{38}$$

this can be written as

$$\Lambda = \left(\frac{m}{m_P}\right)^4 t_P^{-2} \tag{39}$$

Using commonly accepted values, and taking for m the largest mass scale known to exist on the particle level, the ratio of Λ over the square of the Hubble parameter turns out to be 10^{118}. Observers tell us that this number is actually about 1. So, either $\Lambda = 0$ due to some unknown mechanism, or we always have $\Lambda \approx H^2$ due to some equally unknown mechanism.

If the latter proportionality occurs, we must either have Λ evolving with H, for which there is not a shred of theoretical underpinning; or else H must be a true Hubble *constant*, for which there is not a shred of astronomical evidence. But let me persist: if $\Lambda \approx H^2$, then we can use the connection of H with cosmic time, $H \approx 1/t$, to write

$$\left(\frac{m}{m_P}\right)^4 \left(\frac{t_P}{t}\right)^2 \approx 1 \tag{40}$$

which means that the mass m, which now plays the role of an absolute 'vacuum mass scale' must obey

$$m = m_P \sqrt{\frac{t_P}{t}} \approx 10^{-7}\, m_e \tag{41}$$

at the present time. Do we have a candidate particle with one-millionth of the mass of the electron? That is unknown, because its visibility depends on its interactions, which are yet more speculative than the above. And even if Eq.(41) were valid, it would run into difficulties at very early times. If we equate the Compton length of m to the horizon radius,

$$\frac{\hbar}{mc} = \frac{c}{H} \tag{42}$$

one immediately derives

$$m = (Ht_P)m_P = m_P\, \frac{t_P}{t} \tag{43}$$

This implies that before $t \approx t_P$, the mass m would be smaller than this limit, so that its Compton length would be bigger than the local horizon.

The Universe is made of particles, space and time. Since Einstein we know that spacetime is real stuff. But what is it made of? If spacetime is somehow made of particles (whatever those are), how does a photon propagate, i.e. interact with whatever it is that spacetime is made of? How can a graviton, which presumably is a spacetime structure, propagate through space and time? In general: if space has microstructure, what precisely does it mean to say that a particle moves through space? The answer to this question must somehow be related to the old chestnut: why don't particles expand with the Universe?

Figure 8. My installation in the entry hall of the J.H. Oort Building in Leiden, showing a portrait of Jan Oort at 80 years, flanked by images of the Crab Nebula, Comet Halley, and a hydrogen channel map of the Milky Way. The streaks on the left are water damage due to construction flaws in the new building.

The most elementary quantum relations, the De Broglie conditions, say that energy is proportional to frequency and therefore inversely proportional to length. From the redshift we know that the quantum length of photons couples directly to the scale of the Universe, but then why doesn't the Compton length do the same?

9. Conclusion

So much, then, for my ramblings on "Modern developments in historical cosmology." Historians put a lot of emphasis on the published literature. But for me, history is what actually happened, rather than what found its way into print. Even the events in a small corner of the field can be totally absorbing for the soldier who happens to be fighting there.

When the battle is over, the generals begin to write their memoirs. But in science, the battle always goes on. Moreover, there is not really a fight. It is a quest, a journey, and even though the old generals can be very nasty to each other, it is rare for anyone to be seriously hurt.

Which brings me back to Jan Oort. I am working in a brand-new building in Leiden that bears his name. I designed a work of art for the entry hall (Fig.8). The Jan Hendrik Oort Building was 'designed' by a modern architect, the sort of person who, as they say in American politics, is a major-league **bleep**. So it is leaking on all sides, and even the walls are literally falling down, three years after its construction. I think that this is emblematic of big, modern science, and it makes me deeply sad that this is so.

Therefore, in defiance, I dedicate this writing to the memory of that great man who was born 100 years ago, last April. After all, Oort never wrote his memoirs.

He just wrote history.

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