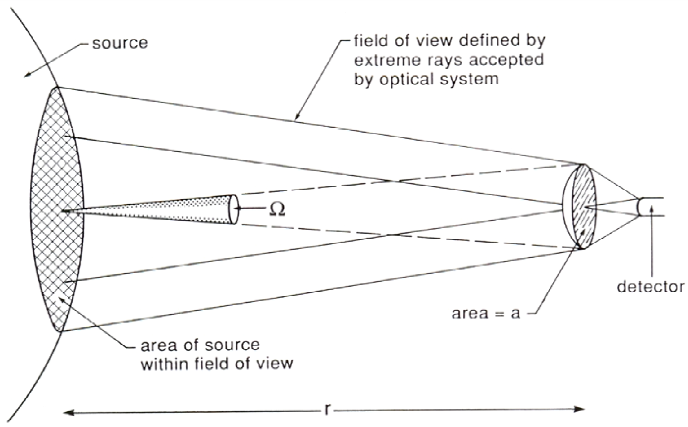
Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

1st Lecture continued: 14 September 2011



This lecture:

- Coherence of Light
- Polarization

5. Coherence of Light

Coherence (from Latin *cohaerere* = to be connected) of EM waves enables temporally and spatially constant interference.

Best case of an uni-directional monochromatic wave (perfect laser): it is possible to define the relative phase at two arbitrary points along **k**.

Worst case (in terms of coherence): black-body radiation.

Two types of coherence:

1. spatial coherence → image formation
2. temporal coherence → spectral analysis

First we consider the wave aspect of light...

Degree of Coherence

Consider a complex field V(t) as a stationary random process with time average <V(t)> = 0.

Measure the fields at any two points in space $V_1(t)$ and $V_2(t)$. The cross correlation between these measurements is given by

$$\Gamma_{12}(\tau) = \langle V_1(t) V_2^*(t+\tau) \rangle$$

whereas the *mean* intensity at point 1 can be described by

$$\Gamma_{11}(0) = \langle V_1(t) V_1^*(t) \rangle$$

The degree of coherence can then be defined as:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{[\Gamma_{11}(0)\Gamma_{22}(0)]^{1/2}}$$

Note that γ_{12} includes both *spatial* (points 1,2) and *temporal* (τ) coherence.

Quasi-Monochromatic Radiation

It can be shown that the relation between spectral width $\Delta\nu$ and temporal width τ_c is:

$$\tau_c \Delta\nu \cong 1$$

The coherence length l_c is the length over which the field retains the memory of its phase.

$$l_c = c\tau_c = \frac{\lambda_0^2}{\Delta\lambda}$$

It is the distance beyond which the waves λ and λ+Δλ are out of step by λ:

For $l \ll c\tau_c$ it follows that: $\gamma_{12}(\tau) \sim \gamma_{12}(0) e^{-2i\pi\nu_0\tau}$

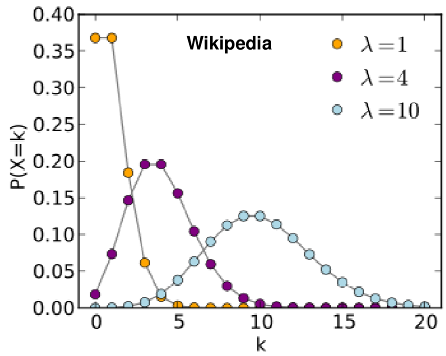
For purely monochromatic radiation, τ is infinite.

Next we will consider the particle aspect of light...

Photon Statistics (1): Poissonian

For a constant classical intensity, the probability of a detected photon event will follow a Poissonian distribution: $f(k;\lambda)=\frac{\lambda^k e^{-\lambda}}{k!}$

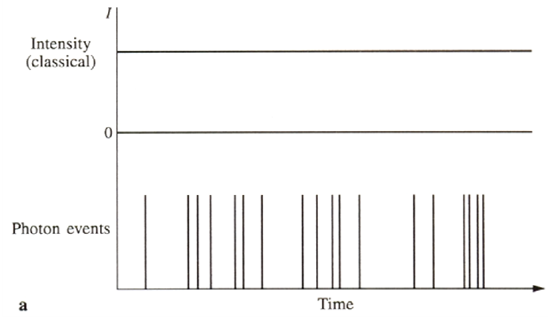
where k is the number of actual occurrences of the event, and λ is the expected number of occurrences (e.g., mean intensity)



In this case, for any time τ the variance and standard deviation are given by (here $\lambda \rightarrow \bar{n}$):

$$\langle \Delta n^2 \rangle = \bar{n}\,\tau$$

$$\sigma = \sqrt{\bar{n}\,\tau}$$



Photon Statistics (2): Bose-Einstein

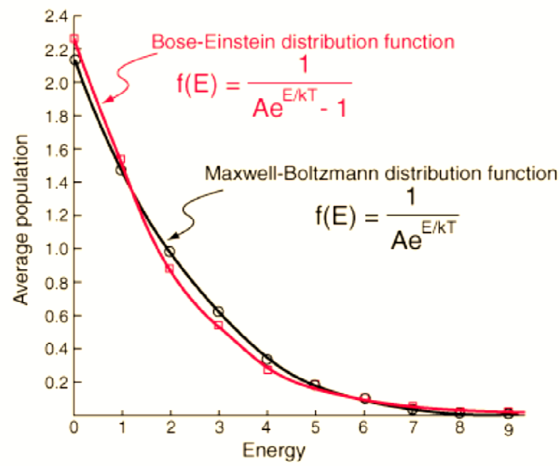
Quasi-monochromatic radiation (e.g., a spectral line) with finite coherence time $\tau_c \sim 1/\Delta\nu$ (where $\Delta\nu$ is the line width).

If $\tau \gg \tau_c$ (non-thermal radiation) the photon fluctuation is affected by the Bose-Einstein distribution:

$$f(E)=\frac{1}{Ae^{E/kT}-1}$$

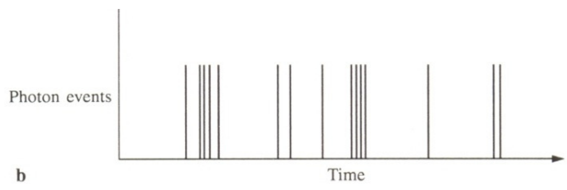
with a variance of:

$$\langle \Delta n^2 \rangle = \bar{n}\,\tau\left(1+\frac{1}{e^{E/kT}-1}\right)$$



Photon Statistics (3): Bunching

- Statistical tendency for multiple photons to arrive simultaneously



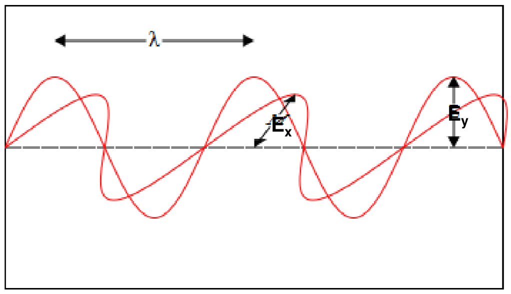
- A property of all bosons (fermions show the opposite effect due to the Pauli exclusion principle)

- Experimentally known as Hanbury-Brown and Twiss effect (→intensity interferometer) → R.J. Glauber, Nobel Prize 2005

6. Polarization of Light

The wave vectors of the electric field are given by:

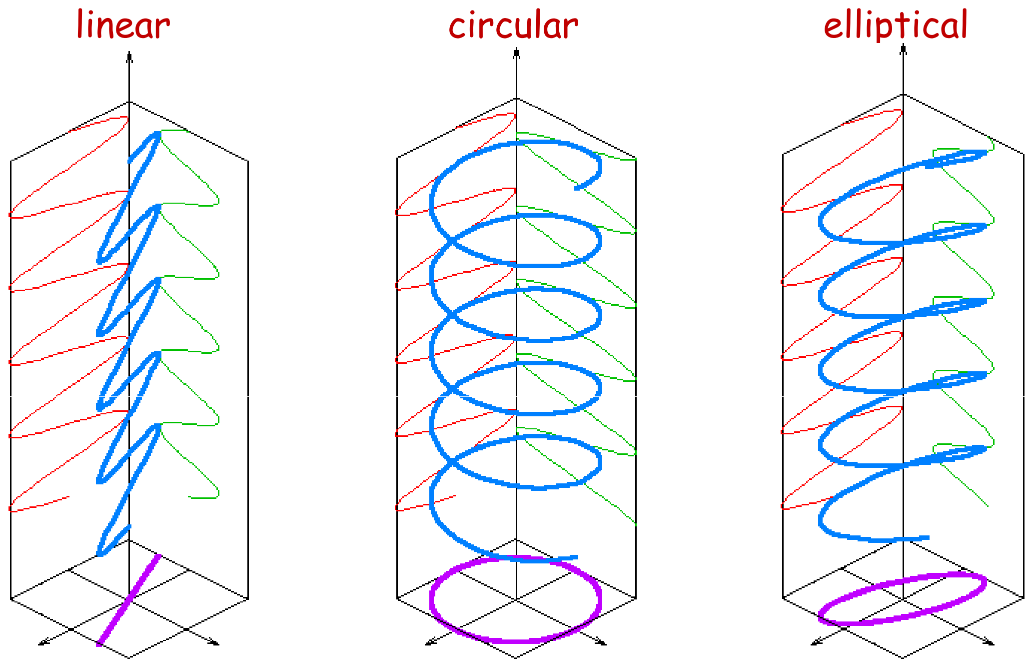
$$E_x = a_1 \cos(2\pi\nu t - k \cdot r + \phi_1)$$
$$E_y = a_2 \cos(2\pi\nu t - k \cdot r + \phi_2)$$



where a_i are the amplitudes, ν is the frequency, k=2π/λ the wavevector, and Φ_i are the phases.

We also define $\Phi = \Phi_2 - \Phi_1$

Three Types of Polarized Waves



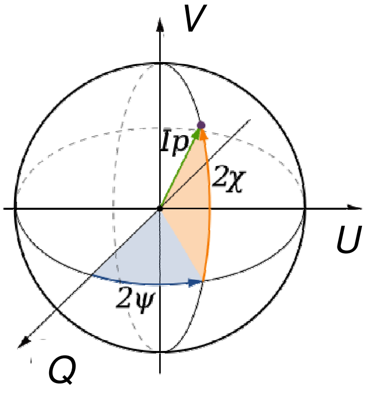
Type and degree of polarization is important as it carries information on the properties of the source (magnetic fields, dust grain alignment, etc.).

Parameter set: Intensity I, degree of polarization Π, ellipse parameters a_1, a_2

The Stokes Parameter

Polarization is best defined by the four Stokes parameters I, Q, U, V (1852) as follows:

$$I = a_1^2 + a_2^2$$
$$Q = a_1^2 - a_2^2 = I\cos 2\chi \cos 2\psi$$
$$U = 2a_1a_2\cos\phi = I\cos 2\chi \sin 2\psi$$
$$V = 2a_1a_2\sin\phi = I\sin 2\chi$$



Generally, the degree of polarization of a wave is:

$$\Pi = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

A plane wave has Π = 1 and the Stokes parameters are related as:

$$I^2 = Q^2 + U^2 + V^2$$

Examples

Polarizers can be used to filter out e.g., reflected light