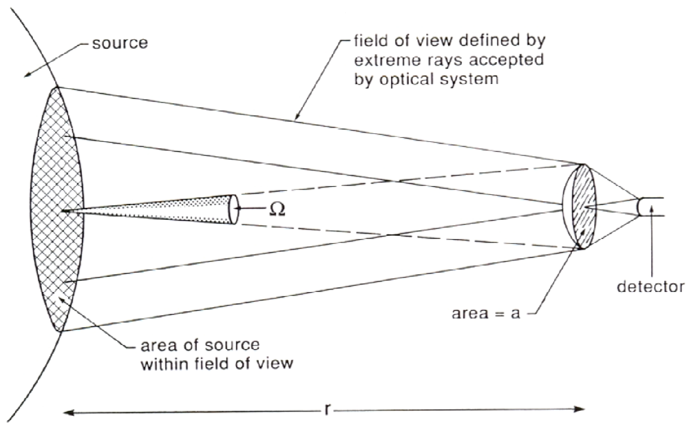
Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

1st Lecture: 12 September 2011



This lecture:

- Radiometry
- Radiative transfer
- Black body radiation
- Astronomical magnitudes

Preface: The Solid Angle

Babylonians: one degree = 1/360th of a full circle

Better measure: θ = (arc length s) / (radius of the circle r) in radians

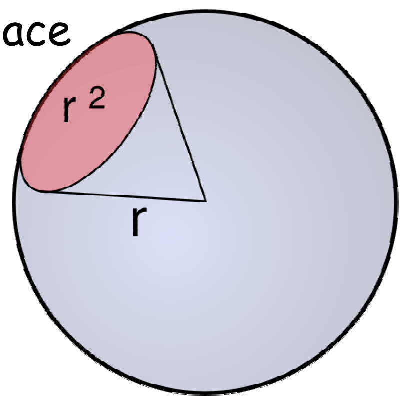
In three dimensions, the solid angle Ω [measured in steradians] is the 2D angle that an object subtends at a point, corresponding to the area it cuts out:

$$\Omega = \text{(surface area S)} / \text{(radius of the sphere r)}$$

One steradian is the solid angle at the center of a sphere of radius r under which a surface of area r^2 is seen.

A complete sphere = 4π sr.

1 sr = $(180\text{deg}/\pi)^2$ = 3282.80635 deg^2



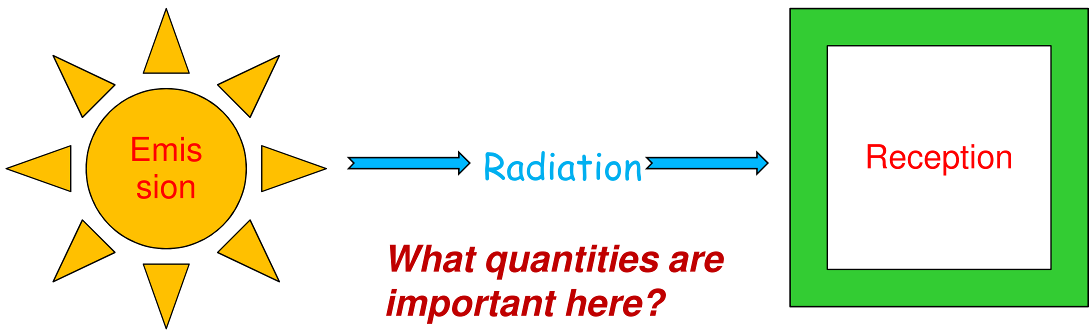
1. Radiometry

Radiometry = the physical quantities associated with the energy transported by electromagnetic radiation.

Photon energy: $E_{ph} = h\nu = \frac{hc}{\lambda}$

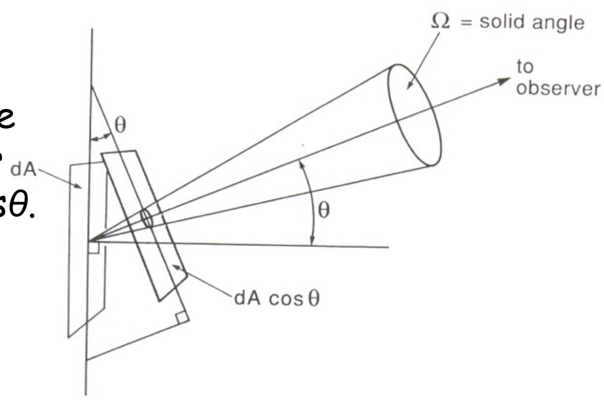
with h = Planck's constant [$6.626 \cdot 10^{-34}$ Js]

See also
http://web.archive.org/web/20080527160217/www.optics.arizona.edu/Palmer/rpfaq/rpfaq.htm



Emission (1): Radiance L or Intensity I

Consider a projected area of a surface element dA onto a plane perpendicular to the direction of observation dA cosθ. θ is the angle between both planes.

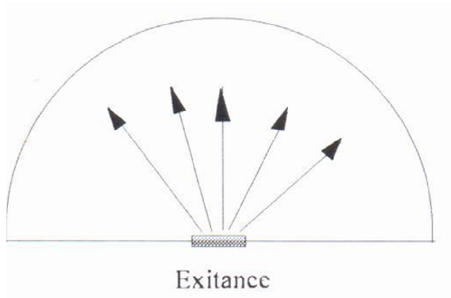


The spectral radiance L_ν or specific intensity I_ν is the power leaving a unit projected area [m^2] into a unit solid angle [sr] and unit bandwidth. Units are:

- [W m^{-2} sr^{-1} Hz^{-1}] in frequency space, or
- [W m^{-3} sr^{-1}] in wavelength space → L_λ

The radiance L or intensity I is the spectral radiance integrated over all frequencies or wavelengths.
Units are [W m^{-2} sr^{-1}].

Emission (2): Exitance or Total Power M



The radiant exitance M is the integral of the radiance over the solid angle Ω.

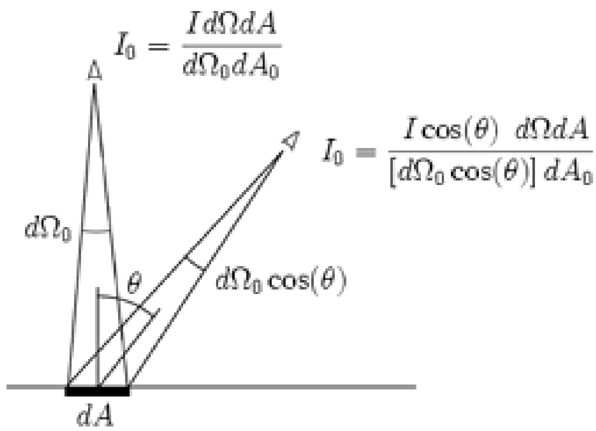
It measures the total power emitted per unit surface area.

Units are [W m^{-2}].

Lambertian Emitters

The radiance of Lambertian emitters is independent of the direction θ of observation (i.e., isotropic).

When a Lambertian surface is viewed from an angle θ, then $d\Omega$ is decreased by cos(θ) but the size of the observed area A is increased by the corresponding amount.



Example: the Sun is almost a perfect Lambertian radiator (except for the limb) with a uniform brightness across the disk.

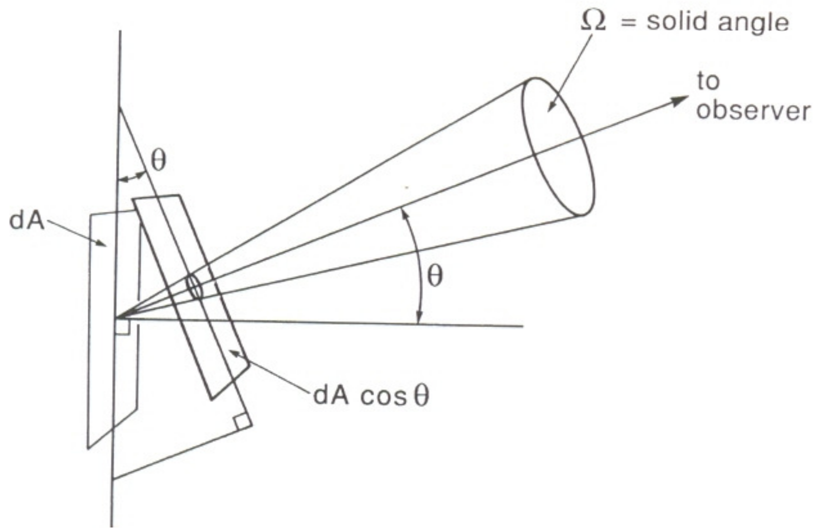
Perfect black bodies obey Lambert's law (1760)

Johann Heinrich Lambert (1728 – 1777)

Emission (2b): Exitance or Total Power M

For Lambertian sources we get:

$$M = \int L(\theta) d\Omega = \int L \cos\theta d\Omega = 2\pi L \int_0^{\pi/2} \sin\theta \cos\theta d\theta \overset{\int \sin\theta\cos\theta d\theta = \frac{\sin^2\theta}{2}}{=} \pi L$$



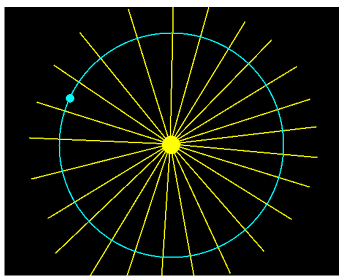
Emission (3): Flux Φ and Luminosity L

The flux Φ or luminosity L emitted by the source is the product of radiant exitance and total surface area of the source.

Note that the luminosity 'L' is different from the radiance 'L' !?!

It is the total power emitted by the *entire* source.

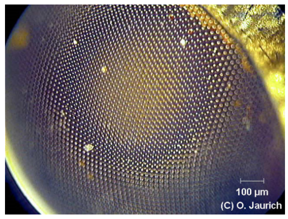
Units are [W] or [erg s^{-1}]



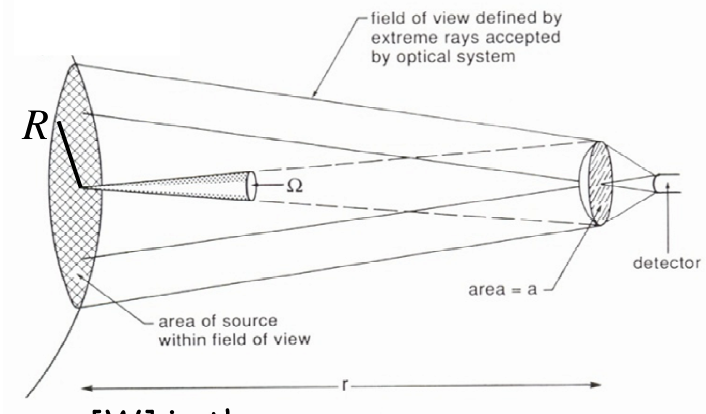
Example: a star of radius R has:

$$\Phi = 4\pi R^2 M \overset{M=\pi L}{=} 4\pi^2 R^2 L$$

The Field of View (FOV)



A detector system usually accepts radiation only from a limited range of directions, determined by the geometry of the optical system, the field of view (FOV).



The *received* power [W] is then:

Radiance *L* [W m^{-2} sr^{-1}] in the direction of our system

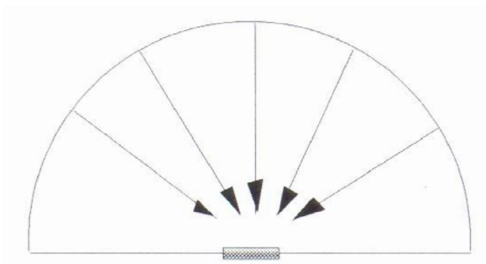
×

Solid angle *Ω* [sr] subtended by our entrance aperture $\Omega = \frac{a}{r^2}$

× (as viewed from source)

Source area *A* [m^2] within our detector FOV $A = \pi R^2$

Reception (1): the Irradiance E



The irradiance E is the power received at a unit surface element from the source. Units are [W m^{-2}].

E can be easily computed:

$$E = \frac{\Phi}{4\pi r^2} = \frac{AM}{4\pi r^2} = \frac{A\pi L}{4\pi r^2} = \frac{AL}{4r^2}$$

Reception (2): the Flux Density F_ν

The spectral irradiance E_ν or flux density F_ν is the irradiance per unit frequency or wavelength interval:

$$F_\nu = \frac{AL_\nu}{4r^2}$$

Units are [W m^{-2} Hz^{-1}] in frequency space or
[W m^{-3}] in wavelength space.

Note: 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1}cm^{-2} Hz^{-1} is also called 1 Jansky, after the US radio astronomer Karl Guthe Jansky.

Karl Guthe Jansky (1905 – 1950)

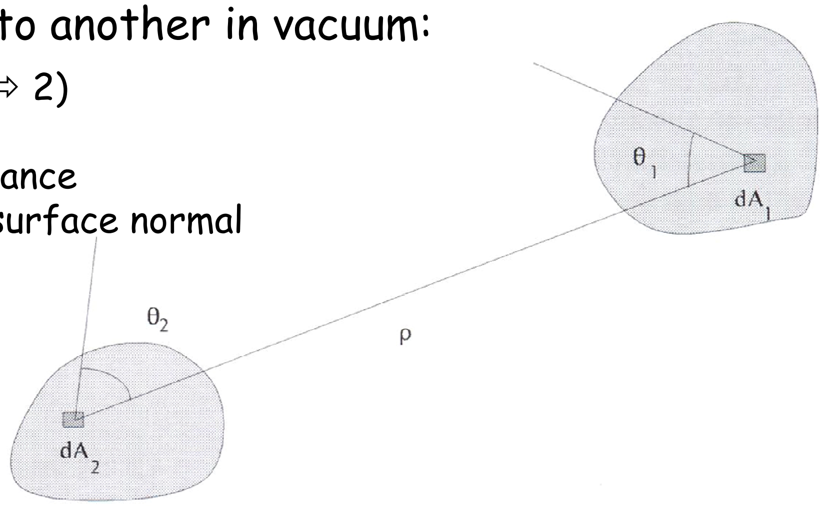
Summary of Radiometric Quantities

Name	Symbol	Unit	Definition	Equation
Spectral radiance *or* specific intensity	L_ν , I_ν	$W\ m^{-2}\ Hz^{-1}\ sr^{-1}$	Power leaving unit projected surface area into unit solid angle and unit $\Delta\nu$	
Spectral radiance *or* specific intensity	L_λ , I_λ	$W\ m^{-3}\ sr^{-1}$	Power leaving unit projected surface area into unit solid angle and unit $\Delta\lambda$	
Radiance *or* Intensity	L , I	$W\ m^{-2}\ sr^{-1}$	Spectral radiance integrated over spectral bandwidth	$L = \int L_\nu\, d\nu$
Radiant exitance	M	$W\ m^{-2}$	Total power emitted per unit surface area	$M = \int L(\theta)\, d\Omega$
Flux *or* luminosity	Φ , L	W	Total power emitted by a source of surface area A	$\Phi = \int M\, dA$
Spectral irradiance *or* flux density	L_ν, F_ν, I_ν	$W\ m^{-2}\ Hz^{-1}$	Power received at a unit surface element per unit $\Delta\nu$	
Spectral irradiance *or* flux density	$L_\lambda, F_\lambda, I_\lambda$	$W\ m^{-3}$	Power received at a unit surface element per unit $\Delta\lambda$	
Irradiance	E	$W\ m^{-2}$	Power received at a unit surface element	$E = \frac{\int M\, dA}{4\pi r^2}$

2. Radiative Transfer

Fundamental equation to describe the transfer of radiation from one surface to another in vacuum:

L – net radiance (1 ⇔ 2)
$A_{1,2}$ – areas
ρ – line of sight distance
$\theta_{1,2}$ – angles between surface normal and line of sight



Then the transferred flux Φ is:

$$d\Phi = L\frac{dA_1 \cos\theta_1 dA_2 \cos\theta_2}{\rho^2}$$

Differential Throughput and Etendue

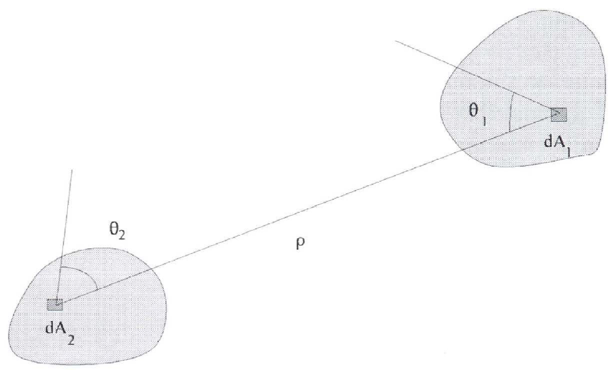
Using the definition of the solid angle $d\Omega_{12} = \frac{dA_1 \cos\theta_1}{\rho^2}$ one can show that: $d\Phi = LdZ$

where *dZ* is the differential throughput.

dZ is also called the étendue (extent, size) or the A×Ω product. Units are [m^2 sr]

Note:

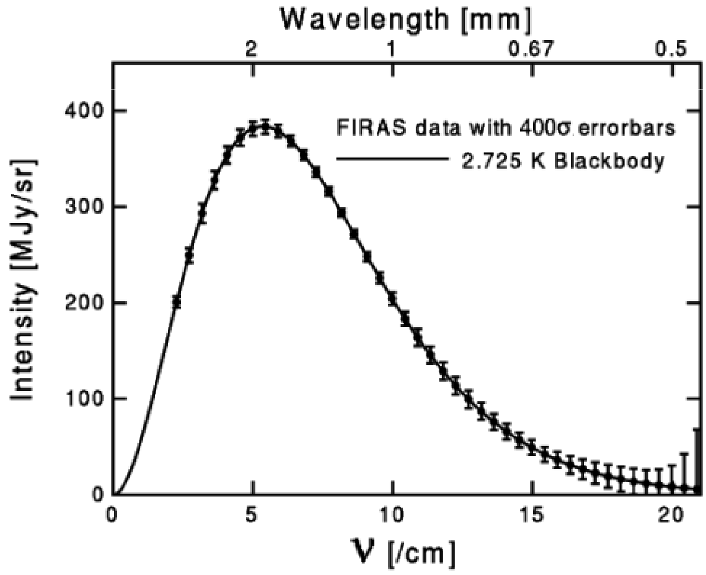
- *L is property of the source,*
- *dZ is a property of the geometry.*



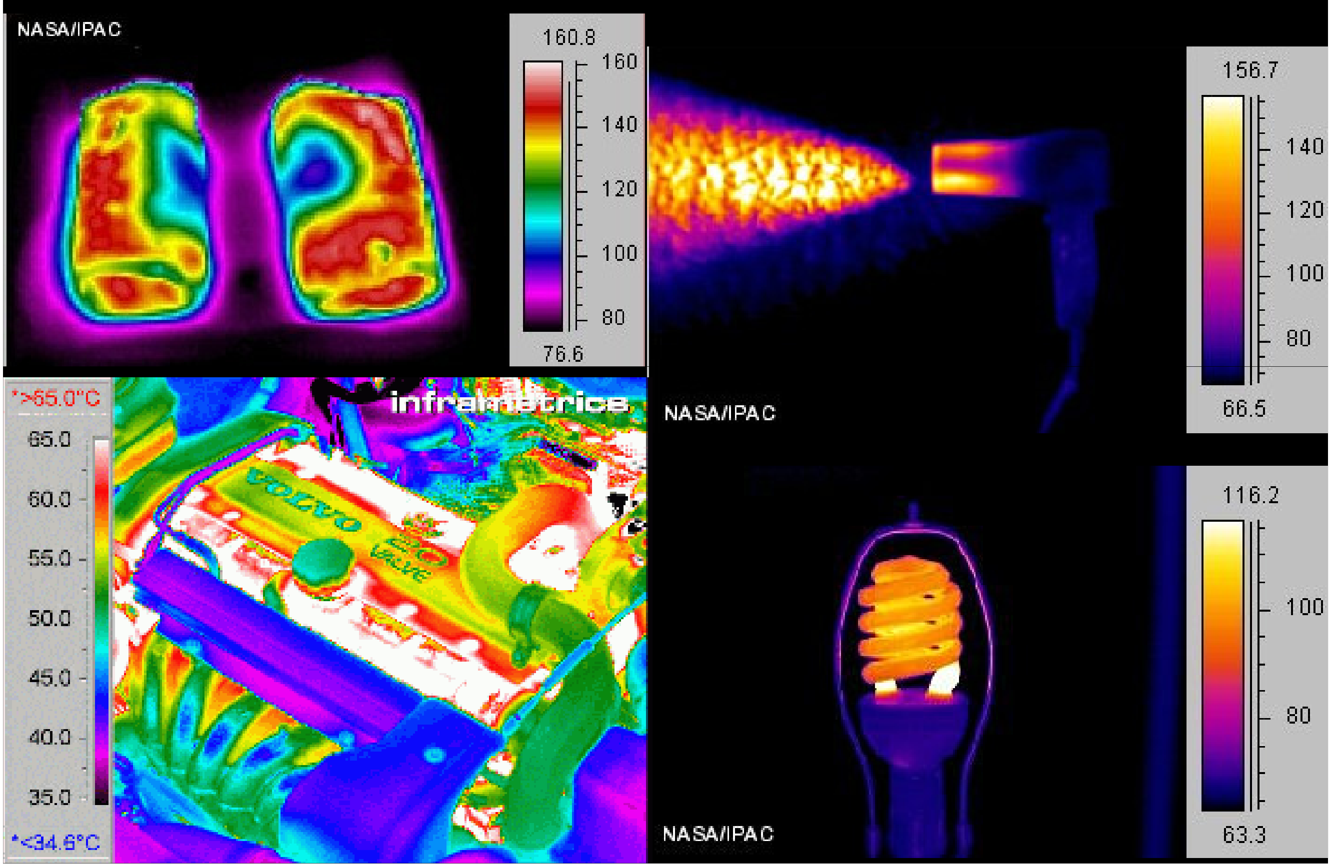
3. Black Body Radiation

- A black body (BB) is an idealized object that absorbs all EM radiation
- Cold (T~0K) BBs are black (no emitted or reflected light)
- At T > 0 K BBs absorb and re-emit a characteristic EM spectrum
- Many astronomical sources emit close to a black body.

Example:
COBE measurement of the cosmic microwave background



Temperature ⇔ Radiation



Black Body Emission

The specific intensity I_ν of a blackbody is given by Planck's law as:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$ in units of [W m^{-2} sr^{-1} Hz^{-1}]

In terms of wavelength units this corresponds to:

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$ in units of [W m^{-3} sr^{-1}]

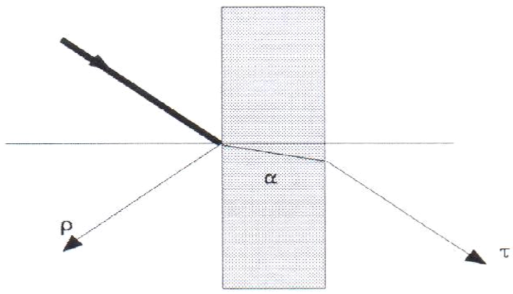
Note for the conversion of frequency ⇔ wavelength units:

$$d\nu = \frac{c}{\lambda^2} d\lambda \quad \text{or} \quad d\lambda = \frac{c}{\nu^2} d\nu$$

Kirchhoff's Law

Conservation of power requires that:

$$\alpha + \rho + \tau = 1$$



with α = absorptivity, ρ = reflectivity, τ = transmissivity

Consider a cavity in thermal equilibrium with completely opaque sides:

$$\left.\begin{aligned} \varepsilon &= 1 - \rho \\ \alpha + \rho + \tau &= 1 \\ \tau &= 0 \end{aligned}\right\} \quad \boxed{\alpha = \varepsilon}$$

This is Kirchhoff's law, which applies to a perfect black body

A radiator with ε = ε(λ) <~ 1 is often called a grey body

Useful Approximations

At high frequencies ($h\nu \gg kT$) we get Wien's approximation:

$$I_\nu(T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{kT}\right)$$

At low frequencies ($h\nu \ll kT$) we get Rayleigh-Jeans' approximation:

$$I_\nu(T) \approx \frac{2\nu^2}{c^2}kT = \frac{2kT}{\lambda^2}$$

The total radiated power per unit surface (radiant exitance) is proportional to the fourth power of the temperature:

$$\int_\Omega \int_\nu I_\nu(T)\, d\nu\, d\Omega = M = \sigma T^4$$

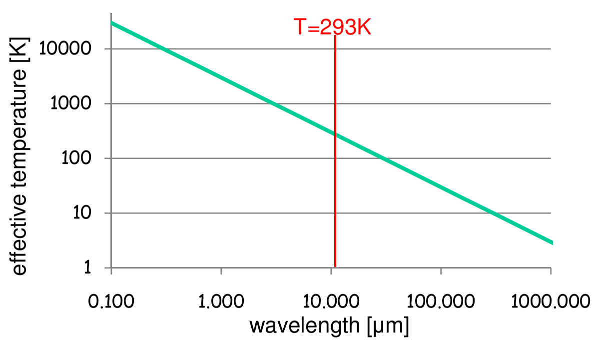
where $\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzmann constant.

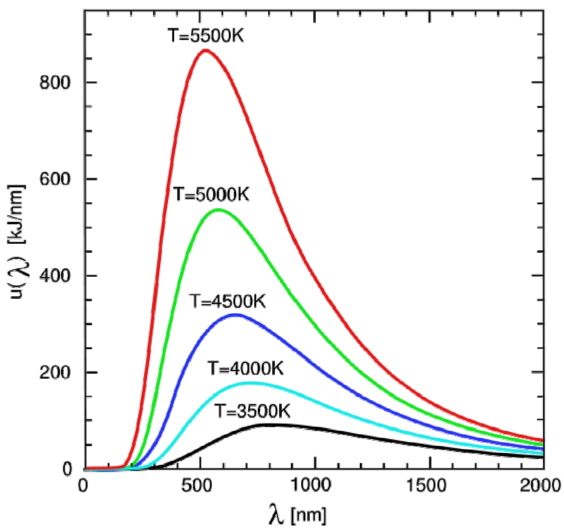
Black Body "Peak" Temperatures

The temperature corresponding to the maximum specific intensity is given by Wien's displacement law:

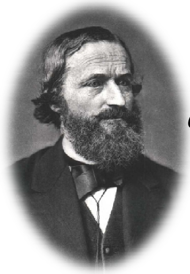
$$\frac{c}{\nu_{max}} T = 5.096 \cdot 10^{-3} \text{ mK} \quad \text{or} \quad \lambda_{max} T = 2.98 \cdot 10^{-3} \text{ mK}$$

Hence, cooler BBs have their peak emission (effective temperatures) at longer wavelengths and at lower intensities:





Assuming BB radiation, astronomers often specify the emission from objects via their effective temperature.



$$\left.\begin{aligned}\varepsilon &= 1-\rho \\ \alpha+\rho+\tau &= 1 \\ \tau &= 0\end{aligned}\right\} \quad \alpha = \varepsilon$$

Gustav Kirchhoff (1824 – 1887)

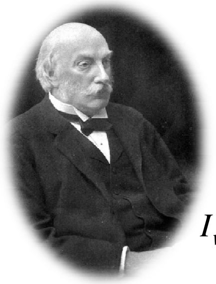
Josef Stefan (1835 – 1893)

$$M = \sigma T^4$$

Ludwig Eduard Boltzmann (1844 – 1906)

$$I_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$$

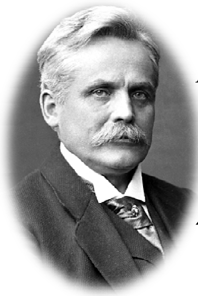
Max Planck (1858 – 1947)



$$I_\nu(T) \approx \frac{2\nu^2}{c^2}kT$$

John William Strutt, 3rd Baron Rayleigh (1842 – 1919)

Sir James Hopwood Jeans (1877 – 1946)



$$I_\nu(T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{kT}\right)$$

$$\lambda_{max} = \frac{2.98\cdot 10^{-3}\ \text{mK}}{T}$$

Wilhelm Wien(1864 – 1928)

4. Magnitudes

This system has its origins in the Greek classification of stars according to their visual brightness. The brightest stars were m = 1, the faintest detected with the bare eye were m = 6.

Later formalized by Pogson (1856): 1^{st} mag ~ 100 × 6^{th} mag

Magnitude	Example	#stars brighter
-27	Sun	
-13	Full moon	
-5	Venus	
0	Vega	4
2	Polaris	48
3.4	Andromeda	250
6	Limit of naked eye	4800
10	Limit of good binoculars	
14	Pluto	
27	Visible light limit of 8m telescopes	

Apparent Magnitude

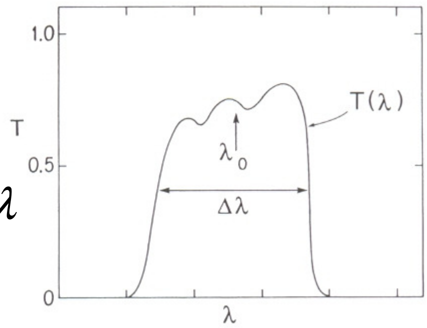
The apparent magnitude is a *relative* measure of the monochromatic flux density F_λ of a source:

$$m_\lambda - M_0 = -2.5 \cdot \log\left(\frac{F_\lambda}{F_0}\right)$$

M_0 defines the reference point (usually magnitude zero).

In practice, measurements are done through a transmission filter $T(\lambda)$ that defines the bandwidth:

$$m_\lambda - M = -2.5\log\int_0^\infty T(\lambda)F_\lambda d\lambda + 2.5\log\int_0^\infty T(\lambda)d\lambda$$



Photometric Systems

Filters are usually matched to the atmospheric transmission
→ different observatories = different filters
→ many photometric systems:

- Johnson UBV system →
- Gunn griz
- USNO
- SDSS
- 2MASS JHK
- HST filter system (STMAG)
- AB magnitude system
- ...

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]
U	0.36	0.068
B	0.44	0.098
V	0.55	0.089
R	0.70	0.22
I	0.90	0.24
J	1.25	0.30
H	1.65	0.35
K	2.20	0.40
L	3.40	0.55
M	5.0	0.3
N	10.2	5
Q	21.0	8

http://en.wikipedia.org/wiki/Photometric_system

Special Magnitude Systems

For a given flux density F_ν, the AB magnitude is defined as:

$$m(AB) = -2.5 \cdot \log F_\nu - 48.60$$

- an object with constant flux per unit frequency interval has zero color.
- zero points are defined to match the zero point of the Johnson V-band
- used by SDSS and GALEX

Similarly, the STMAG system is defined such that an object with constant flux per unit wavelength interval has zero color.

- STMAGs are used by the HST photometry packages.

Color Indices

Color index = difference of magnitudes at different wavebands = ratio of fluxes at different wavelengths.

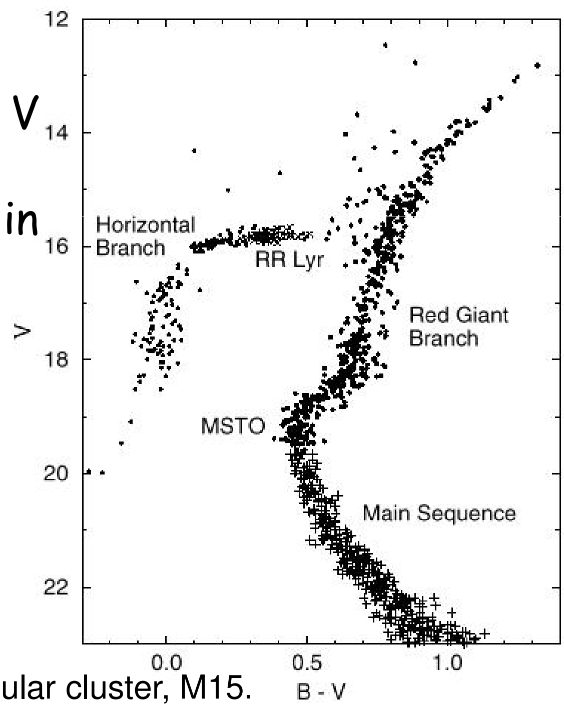
- The color indices of an A0V star (Vega) are about zero longward of V

- The color indices of a blackbody in the Rayleigh-Jeans tail are:

  B-V = -0.46

  U-B = -1.33

  V-R = V-I = ... = V-N = 0.0



Color-magnitude diagram for a typical globular cluster, M15.

Absolute Magnitude

Absolute magnitude = apparent magnitude of the source if it were at a distance of D = 10 parsecs: $M = m + 5 - 5\log D$

M_{Sun} = 4.83 (V); $M_{Milky\ Way}$ = −20.5 → Δmag = 25.3 → Δlumi = 14 billion L_o

However, interstellar extinction *E* or absorption *A* affects the apparent magnitudes

$$E(B-V) = A(B) - A(V) = (B-V)_{\text{observed}} - (B-V)_{\text{intrinsic}}$$

Hence we need to include absorption to get the correct absolute magnitude:

$$M = m + 5 - 5\log D - A$$

Bolometric Magnitude

Bolometric magnitude is the luminosity expressed in magnitude units = integral of the monochromatic flux over all wavelengths:

$$M_{bol} = -2.5 \cdot \log \frac{\int_0^\infty F(\lambda) d\lambda}{F_{bol}} \quad ; F_{bol} = 2.52 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2}$$

If the source radiates isotropically one can simplify:

$$M_{bol} = -0.25 + 5 \cdot \log D - 2.5 \cdot \log \frac{L}{L_\Theta} \quad ; L_\Theta = 3.827 \cdot 10^{26}\ \text{W}$$

The bolometric magnitude can also be derived from the visual magnitude plus a bolometric correction BC:

$$M_{bol} = M_V + BC$$

BC is large for stars that have a peak emission very different from the Sun's.

Photometric Systems and Conversions

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]	F_λ [W m^{-2} μm^{-1}]	F_ν [Jy]	
U	0.36	0.068	4.35×10^{-8}	1 880	Ultraviolet
B	0.44	0.098	7.20×10^{-8}	4 650	Blue
V	0.55	0.089	3.92×10^{-8}	3 950	Visible
R	0.70	0.22	1.76×10^{-8}	2 870	Red
I	0.90	0.24	8.3×10^{-9}	2 240	Infrared
J	1.25	0.30	3.4×10^{-9}	1 770	Infrared
H	1.65	0.35	7×10^{-10}	636	Infrared
K	2.20	0.40	3.9×10^{-10}	629	Infrared
L	3.40	0.55	8.1×10^{-11}	312	Infrared
M	5.0	0.3	2.2×10^{-11}	183	Infrared
N	10.2	5	1.23×10^{-12}	43	Infrared
Q	21.0	8	6.8×10^{-14}	10	Infrared

1 Jy = 10^{-26} W m^{-2} Hz^{-1}.

Surface brightness

Historical, magnitudes were units to describe unresolved objects or point sources.

To describe extended objects one uses the surface brightness in units of mag/sr or mag/arcsec^2.

Because magnitudes are logarithmic one cannot simply divide by area to compare surface brightnesses.

$$S = m + 2.5 \cdot \log A$$

Note that surface brightness is constant with distance!