Coordinate Systems

Labels: celestial sphere, celestial equator, + terrestrial sphere, + Equator, vernal equinox, South celestial pole, celestial meridian, right ascension, ecliptic, declination, North celestial pole, inclination

www.visualdictionaryonline.com
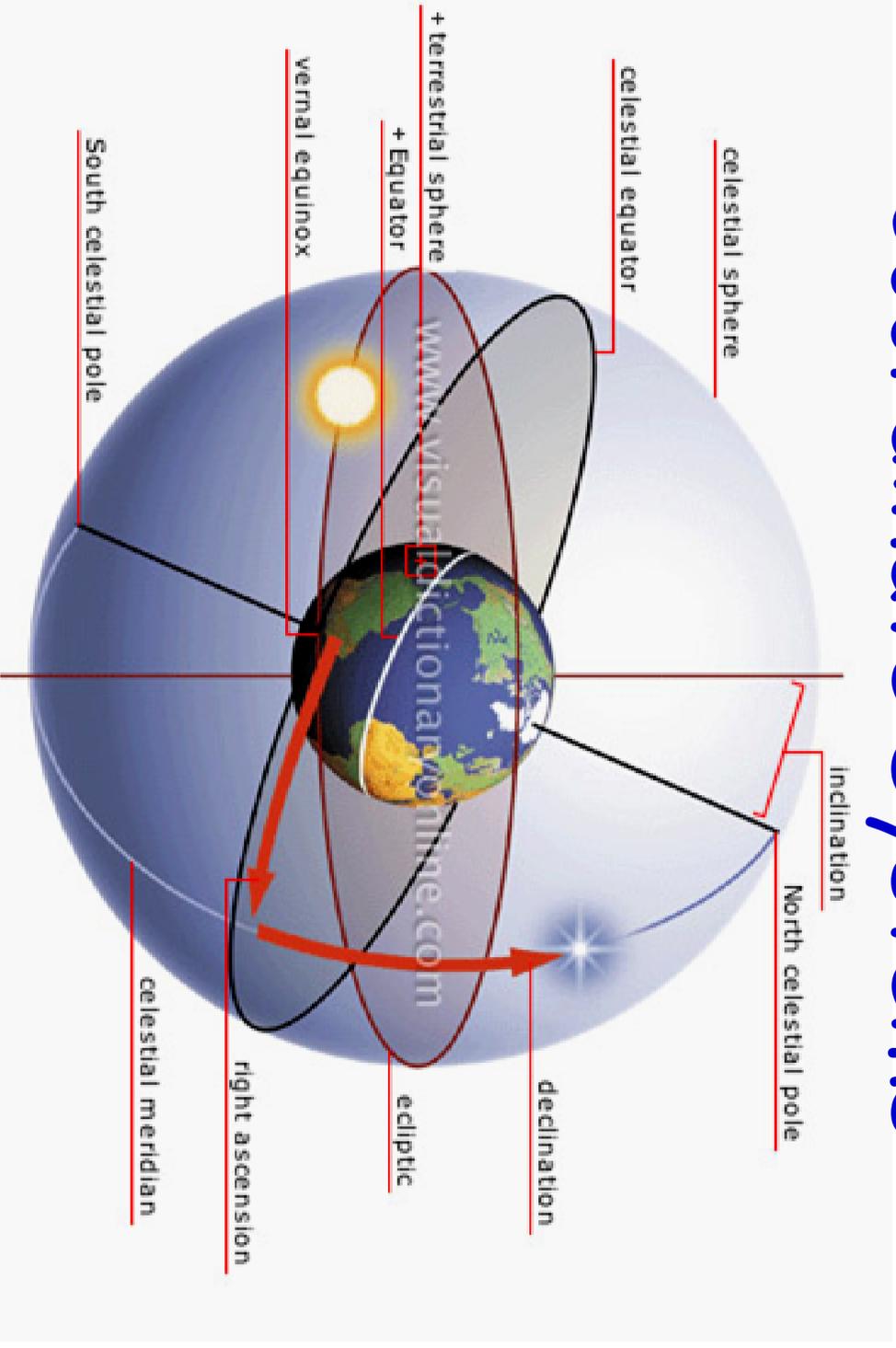
Basis for any 3D Coordinate System

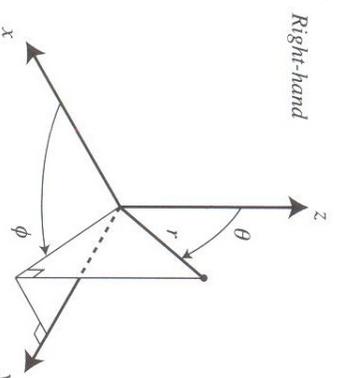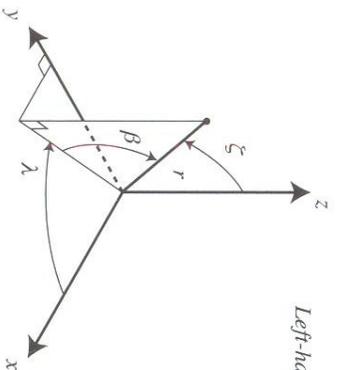Basic steps for the definition of a 3D coordinate system:

1. Locate the origin
2. Locate the x-y plane (the "fundamental plane")
3. Decide on direction of positive x axis ("reference direction")
4. Decide on sign convention of y and z axes

Usual approach to define angles:

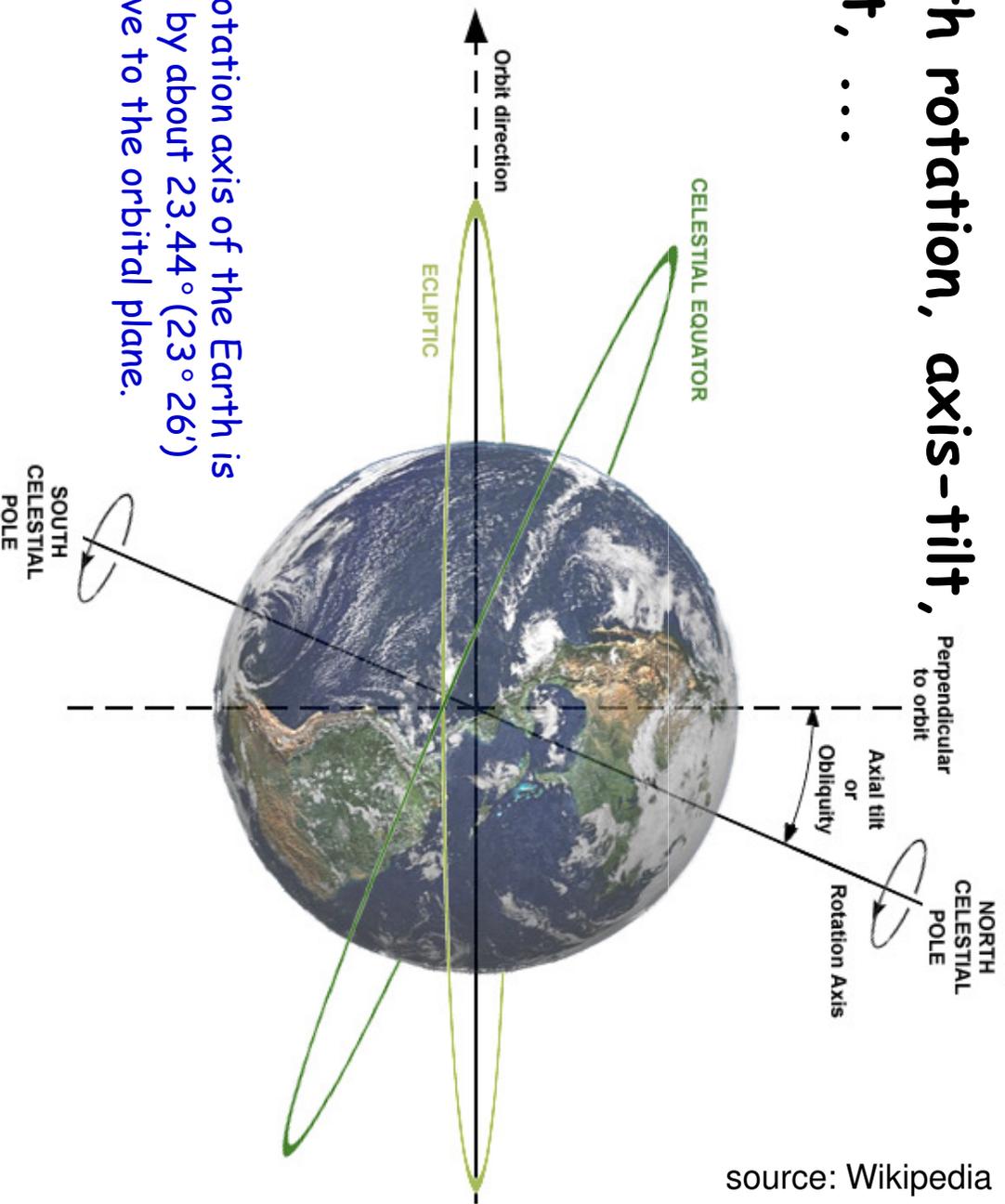- First angle from +x to +y axis [0 ≤ α,φ,λ ≤ 2π]
- Second angle from x-y plane to +z axis [−π/2 ≤ θ,δ,ξ ≤ +π/2]

  or [0 ≤ θ,δ,ξ ≤ π]

Left-hand

Right-hand

Earth rotation, axis-tilt, orbit, ...

The rotation axis of the Earth is tilted by about 23.44°(23°26') relative to the orbital plane.

Definitions (1): the Meridian

A meridian is a great circle on the celestial sphere passing through:

- the north point on the horizon H
- the celestial pole P
- the zenith Z
- the south point on the horizon H'

The meridian is perpendicular to the local horizon.

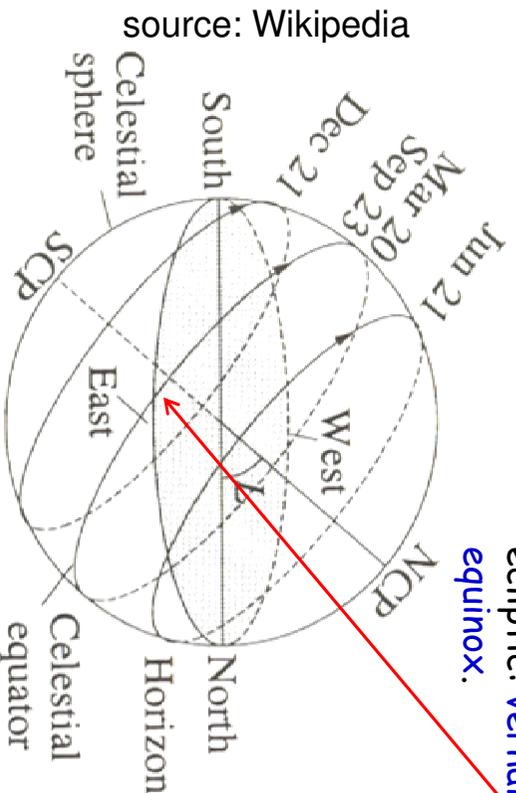Stars will appear to drift past the local meridian as the Earth spins.
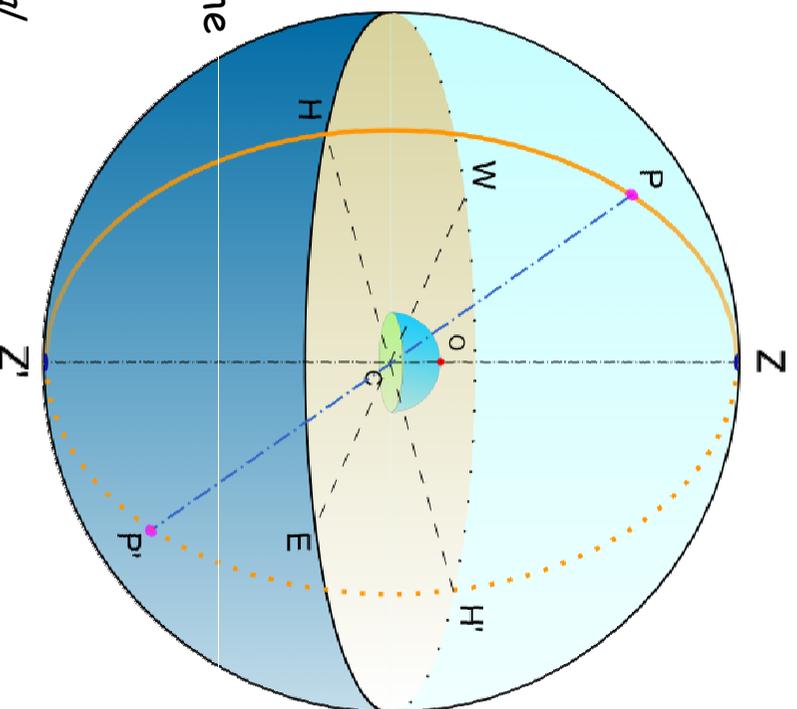
Definitions (2): Equinox

Equinox derives from `aequus' (equal) and `nox' (night).

Equinox = when the centre of the Sun can be observed to be directly above the Earth's equator (i.e., declination δ=0).

At an equinox, the Sun will spend equal time above and below the horizon at every location on Earth.

source: Wikipedia

Intersection point between celestial equator (δ=0) and ecliptic: **vernal/autumnal equinox**.

UTC date and time of solstices and equinoxes[1]								
year		Equinox Mar		Solstice June		Equinox Sept		Solstice Dec
	day	time	day	time	day	time	day	time
2017	20	10:28	21	04:24	22	20:02	21	16:28
2016	20	04:30	20	22:34	22	14:21	21	10:44
2015	20	22:45	21	16:38	23	08:20	22	04:48
2014	20	16:57	21	10:51	23	02:29	21	23:03
2013	20	11:02	21	05:04	22	20:44	21	17:11
2012	20	05:14	20	23:09	22	14:49	21	11:12
2011	20	23:21	21	17:16	23	09:04	22	05:30
2010	20	17:32	21	11:28	23	03:09	21	23:38
2009	20	11:44	21	05:45	22	21:18	21	17:47
2008	20	05:48	20	23:59	22	15:44	21	12:04
2007	21	00:07	21	18:06	23	09:51	22	06:08
2006	20	18:26	21	12:26	23	04:03	22	00:22
2005	20	12:33	21	06:46	22	22:23	21	18:35
2004	20	06:49	21	00:57	22	16:30	21	12:42

Def. (3): Hour Angle and Right Ascension

The **hour angle** is the angle between the meridian plane and the half plane determined by the Earth's axis and the given point. The hour angle is expressed in time units, with one hour = 360°/24hr = 15°.

The **right ascension α** is the angle VOQ between the vernal equinox and the given point on the equator.
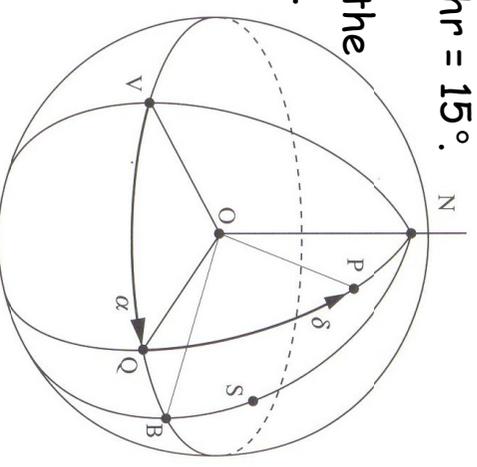
The **hour angle** is also the difference between **local sidereal time (LST)** and **right ascension α**:

$$HA_{object} = LST - \alpha_{object}$$

The object's hour angle indicates how much sidereal time has passed since the object was on the local meridian.
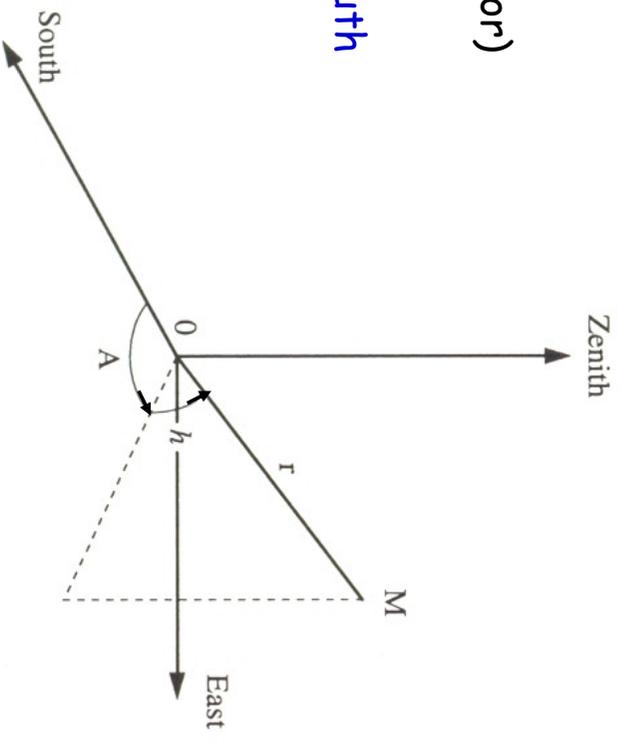
I. The Horizontal Frame (Alt–Azimuth)

Origin: Position of the observer

Axes: local vertical (gravity vector)
horizon

Coordinates: azimuth A from South
height h

$$U(A,h) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos A \cos h \\ \sin A \cos h \\ \sin h \end{bmatrix}$$

Problem: apparent motion of the celestial sphere.

Note: due to tidal forces the gravity vector changes and hence the coordinates relative to surface structures by typically 0.015"

II. The Hour Frame (Latitute–Longitude)

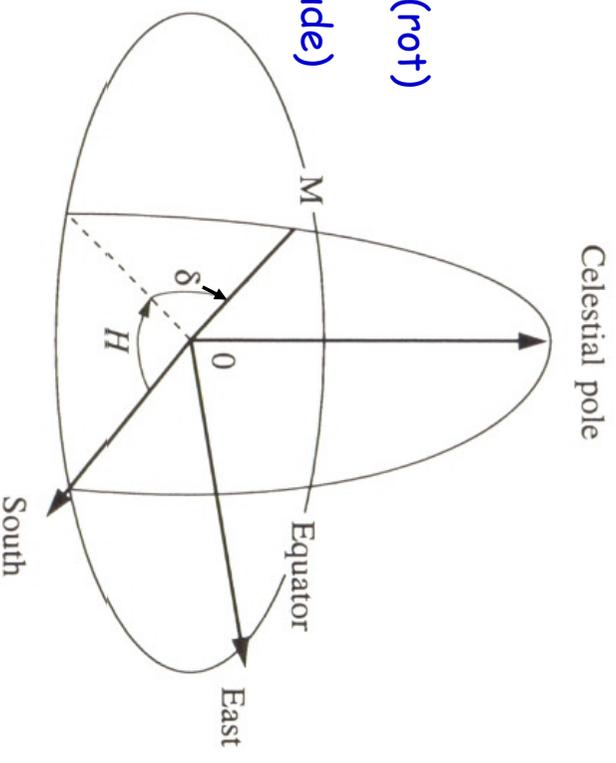Choose celestial equator as fundamental plane.

Origin: Position of the observer

Axes: vector to celestial pole
vector to celestial equator (rot)

Coordinates: declination δ (latitude)
hour angle H

$$U(H,\delta) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos H \cos \delta \\ -\sin H \cos \delta \\ \sin \delta \end{bmatrix}$$

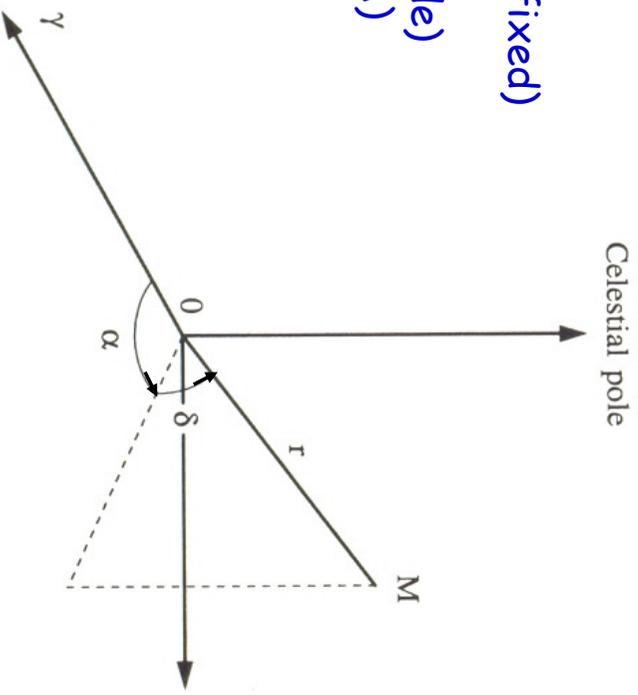Problem: refers to the observer's *local* meridian.

III. The Equatorial Frame

Removes the largest component of Earth rotation

Axes: vector to celestial pole
vector to celestial equator (fixed)

Coordinates: declination δ (latitude)
right ascension α (RA)

$$U(\alpha,\delta) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{bmatrix}$$

The equatorial frame is used for most star and galaxy catalogues.

PPM J2000.0   +80 degrees   Stars No. 2401 ff

PPM	DM	Mag	Sp	R.A. J2000	Dec. J2000	PMA	PMD	N	SA	SD	SPMA	SPMD	EPA	EPD	SAO	HD	AGK3	Notes
01	+88 0084	10.8	G0	13 41 7.059	+87 54 45.48	-0.0022	0.005	9	07	06	2.2	1.9	44.10	43.94	2296		+88 0076	H
02	+84 0316	11.2	F5	13 41 10.613	+83 25 29.66	-0.0091	0.000	4	12	12	5.3	5.4	23.00	24.78	2263		+83 0356	H
03	+81 0445	11.1	G5	13 41 21.575	+81 40.88	-0.0064	0.011	4	12	12	5.4	5.4	24.15	26.00	2261		+81 0417	H
04	+87 0130	10.3	G8	13 41 33.265	+86 35 55.03	-0.0024	-0.021	7	07	07	2.3	2.1	49.34	49.71	2276	120193	+86 0193	H
05	+80 0421	8.5	K5	13 41 39.768	+80 12 13.06	-0.0126	0.001	7	06	07	2.3	2.4	54.13	51.57	2262	120103	+80 0305	H
06	+83 0397	6.0	G5	13 42 23.092	+82 45 8.62	-0.0183	-0.041	0	02	02	0.8	0.7	56.42	40.97	2266	120565	+83 0357	F
07	+81 0446	11.0	F2	13 42 33.663	+80 24 7.97	-0.0045	-0.009	2	02	02	0.8	0.7	25.27	27.25	2264	120565	+83 0306	
08	+81 0447	9.8	K0	13 42 52.566	+80 43 25.36	-0.0113	-0.005	5	10	10	4.4	4.4	23.67	23.85	2265	120363	+80 0307	
09	+88 0085	10.2	K0	13 42 55.608	+87 51 25.03	-0.0695	-0.017	9	09	09	1.9	1.9	44.61	44.34	2301		+80 0077	H
10	+86 0198	11.2	G5	13 43 26.319	+85 42 26.46	-0.0005	-0.024	5	08	09	3.9	4.1	39.32	37.45	2277		+85 0213	
11	+82 0403	11.4	G5	13 43 26.894	+81 32 47.03	-0.0136	-0.026	4	12	12	5.4	5.4	22.53	24.31	2269		+81 0418	
12	+87 0132	12.4	K2	13 44 9.617	+86 50 13.77	-0.0014	-0.004	7	07	07	2.5	2.3	51.11	53.38	2286		+87 0105	H
13	+81 0448	10.7	K0	13 44 25.170	+80 34 24.28	-0.0005	-0.023	4	12	12	5.3	5.2	25.71	26.18	2271		+81 0308	
14	+85 0234	8.8	K0	13 44 31.538	+84 30 34.82	-0.0010	-0.010	8	05	06	2.0	2.0	25.18	26.43	2280	121623	+84 0292	H
15	+86 0199	9.6	F5	13 45 48.743	+85 31 24.16	-0.0121	0.022	7	07	07	3.1	3.1	34.58	31.08	2284	121103	+85 0214	
16	+83 0398	10.8	G5	13 46 27.094	+82 31 46.99	0.0016	-0.029	4	12	11	5.2	5.2	24.71	24.86	2278		+82 0399	
17	+81 0449	10.5	K0	13 47 4.418	+81 16 24.02	0.0065	-0.018	4	11	11	5.1	5.1	26.23	26.70	2279		+81 0419	
18	+84 0317	9.7	F0	13 47 18.812	+84 36 58.58	0.0042	-0.013	9	07	06	4.1	4.1	23.81	23.73	2283		+84 0293	
19	+88 0087	11.2	A5	13 48 12.656	+87 55 34.91	-0.0144	-0.002	9	10	10	2.1	1.9	43.61	45.53	2318		+88 0078	
20	+85 0235	9.8	K2	13 48 18.527	+85 11 55.70	-0.0001	0.003	6	07	08	3.4	3.4	36.98	33.67	2292	122362	+85 0215	
21	+81 0450	11.4	F5	13 48 32.674	+80 21 1.44	0.0012	-0.004	4	12	13	5.6	5.6	24.31	26.35	2281		+81 0421	
22	+86 0201	7.5	F0	13 49 15.571	+85 44 52.23	-0.0370	-0.024	8	05	06	2.3	2.4	52.79	52.75	2298	122882	+86 0216	H
23	+84 0318	10.9	G5	13 49 17.750	+84 57.13	-0.0198	-0.005	8	13	12	5.3	5.3	24.42	24.57	2290		+84 0294	
24	+88 0088	11.2	G0	13 49 51.417	+88 7 39.79	-0.1148	-0.008	8	08	08	3.2	3.2	31.12	29.01	2327		+88 0079	
25	+85 0451	11.2	K0	13 49 55.447	+80 55 14.32	-0.0031	-0.005	4	12	12	5.5	5.5	23.82	25.69	2285		+81 0420	
26	+83 0399	11.5	K0	13 50 1.940	+82 18 47.07	-0.0103	0.010	4	13	12	5.6	5.6	24.65	23.78	2287		+82 0400	
27	+86 0202	10.2	F5	13 50 4.531	+85 56 -01	-0.0106	0.004	4	11	11	2.6	2.6	51.18	50.54	2299		+86 0194	H
28	+83 0401	11.7	G5	13 50 21.839	+82 54 47.92	0.0021	0.017	4	13	12	5.6	5.6	24.51	23.80	2293		+83 0359	H
29	+83 0401	11.7	G0	13 50 22.800	+82 42.81	-0.0021	0.017	4	13	13	5.5	5.5	24.51	23.80	2291		+83 0358	H
30	+81 0452	7.7	K0	13 51 0.570	+80 46 4.28	0.0178	-0.011	6	08	08	3.4	3.4	36.98	52.71	2289	121778	+81 0421	
31	+87 0134	10.9	G0	13 51 27.814	+87 14 27.95	-0.0224	-0.014	7	08	08	3.3	3.2	34.02	30.31	2315		+87 0106	H
32	+84 0320	11.1	G0	13 51 29.795	+84 17.56	-0.0106	0.008	4	12	12	5.3	5.3	34.04	25.10	2298		+84 0295	
33	+89 0031	11.7	G8	13 51 39.885	+89 15 23.97	0.0021	0.000	14	08	06	1.8	1.8	34.24	34.41	2450	126047	+89 0038	
34	+81 0453	11.0	F2	13 51 2.812	+80 48 27.30	0.0124	-0.045	4	12	12	5.3	5.3	24.46	26.30	2295		+89 0422	
35	+82 0404	11.3	G0	13 52 18.733	+81 59 21.03	0.0079	-0.070	4	12	12	5.4	5.4	22.64	24.43	2297		+82 0401	
36	+83 0402	10.0	F8	13 52 29.431	+82 57 1.84	-0.0090	-0.041	5	10	10	4.5	4.3	23.13	23.13	2300		+83 0360	H
37	+87 0133	10.2	G8	13 52 51.384	+86 29 25.25	-0.0033	0.000	8	07	07	3.0	3.0	32.62	28.92	2312		+86 0195	
38	+82 0406	10.2	A5	13 53 53.449	+82 1 59.73	-0.0031	0.004	4	11	11	5.0	4.9	26.58	26.58	2302		+84 0402	
39	+82 0405	10.1	G5	13 54 9.462	+80 48 54.89	-0.0144	-0.020	4	11	11	5.0	5.0	26.67	26.67	2304		+82 0403	
40	+86 0203	12.3	K3	13 54 14.438	+85 25 0.51	0.0039	0.009	5	08	09	4.2	4.4	41.04	39.29	2311		+85 0217	
41	+81 0454	11.1	F5	13 54 30.910	+80 46 37.67	0.0027	-0.008	4	12	12	5.5	5.5	24.15	26.00	2303		+81 0423	
42	+88 0086	8.9	B9	13 54 57.133	+87 23 43.87	0.0062	-0.007	7	07	07	3.2	3.2	34.04	35.94	2326	126047	+89 0108	D
43		12.3		13 55 4.668	+89 49.72	0.0169	-0.010	12	08	07	3.1	3.1	33.21	35.45			+89 0040	
44		12.3	K2	13 55 15.447	+89 24 32.72	-0.0033	-0.006	4	12	12	5.3	5.3	33.24	33.78			+82 0404	H
45	+86 0204	11.6	G5	13 55 18.914	+85 44 14.43	0.0036	-0.004	5	08	09	4.2	4.2	40.73	37.06	2313		+85 0218	
46	+81 0455	11.2	F	13 55 41.642	+80 23 39.53	-0.0119	0.023	4	12	12	5.5	5.7	24.65	26.66	2305		+80 0311	
47	+81 0455	11.2		13 55 41.662														

IV. The Ecliptic Frame

Best for charting solar system objects

Axes: normal to the orbital plane
       vector to vernal equinox

Coordinates: ecliptic lattitude β
              ecliptic longitude λ

Problem: orbital plane (Earth-Sun vector) is sensitive to perturbations by the other planets
→ oscillations of ~1" about mean plane.

Ecliptic pole

Celestial pole

ε

γ

ε

0

V. The Galactic Frame

Useful to chart stellar populations in the MilkyWay.

Origin: Sun

Axes: rotation vector of the Galaxy
       vector to the Galactic Center

Coordinates: galactic latitude b
              galactic longitude l

Definition:

North Galactic pole at 12^h 51^m 26.282^s $+27°$ $07'$ $42.01"$ (J2000)

zero longitude at position angle $122.932°$

zero point is at 17^h 45^m 37.224^s $-28°$ $56'$ $10.23"$ (J2000), close to Sgr A*

90°

0°

Sun

270°

source: Wikipedia

VI. The Supergalactic Frame

Equator aligned with the supergalactic plane formed by the nearby galaxy clusters Virgo, the Great Attractor and the Pisces-Perseus supercluster.

Coordinates: supergalactic lattitude SGB
supergalactic longitude SGL

Zero point for SGL is the intersection of the supergalactic plane with the Galactic plane

The nearest 10 Mpc in SG coordinates
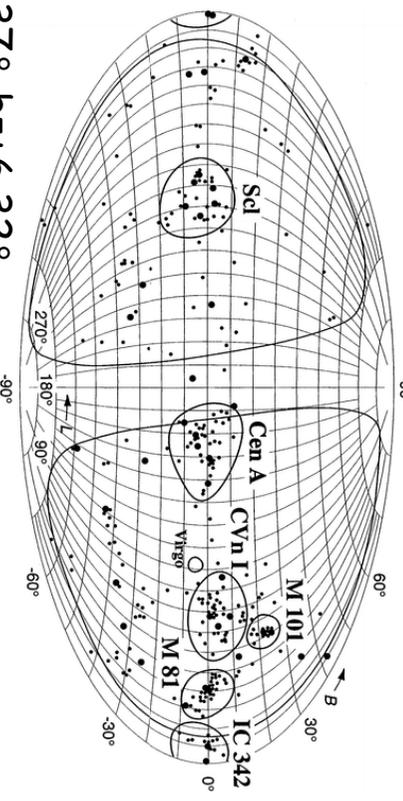
Definition:

North supergalactic pole at l =47.37°, b=+6.32°
or RA=18.9^h, δ=+15.7° (J2000).
Zero point (SGB=0°, SGL=0°) at l=137.37°, b=0°
or RA=2.82^h, δ=+59.5° (J2000).

Coordinate Frame Transformations

The hard way:

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = [R] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \quad R_2(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \quad R_3(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
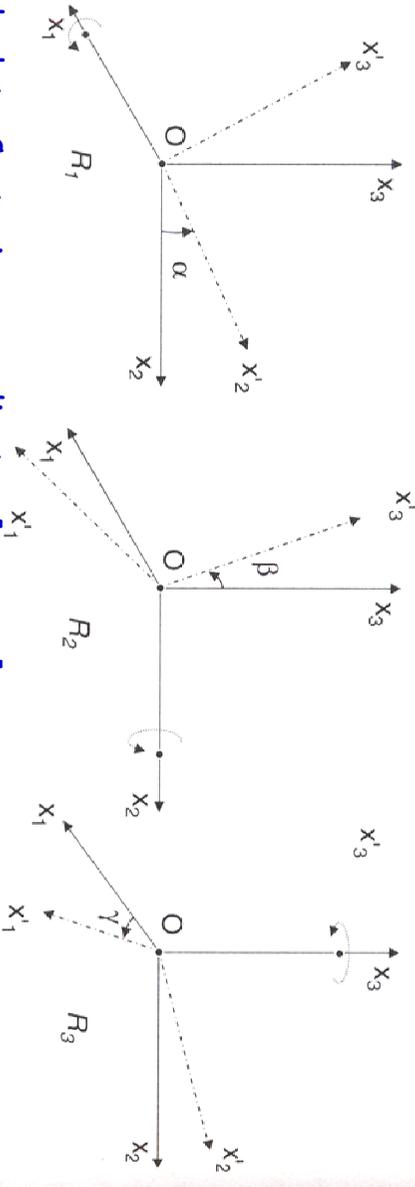
1. Calculate Cartesian coordinates [x_1, x_2, x_3]
2. Apply rotation
3. Calculate transformed frame coordinates

➜ see Monday exercises

Example : from Equatorial to Galactic Frame
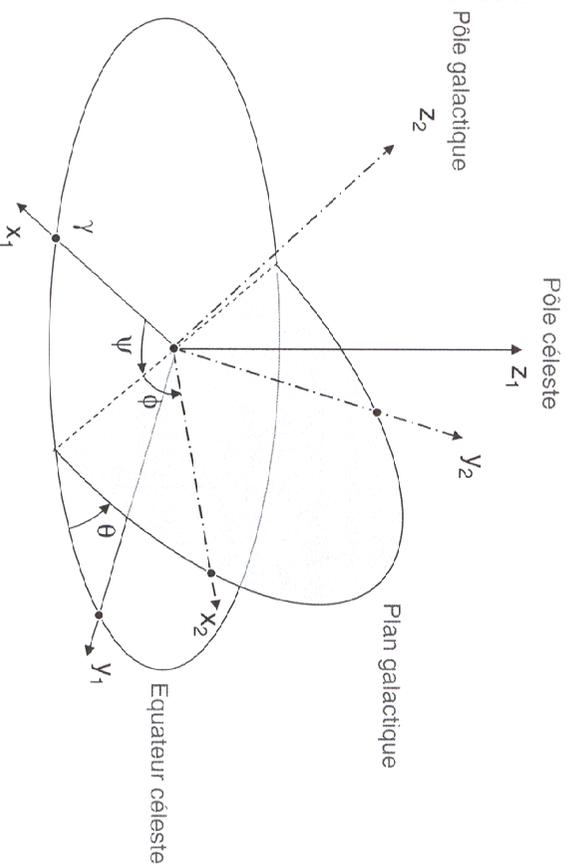
$$U(\lambda, \beta) = R_3(\phi) \cdot R_1(\theta) \cdot R_3(\psi) U(\alpha, \delta)$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} -0.05611 & -0.87374 & -0.48315 \\ 0.49333 & -0.44498 & 0.74741 \\ -0.86803 & -0.19642 & 0.45601 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

See book by Lena for details

Coordinate Frame Transformations (2)

The easy way: http://heasarc.gsfc.nasa.gov/cgi-bin/Tools/convcoord/convcoord.pl

Parallax and Orbital Motion

Parallax = semi-angle of inclination between two sightlines to a star.

Sun

Earth

d

p
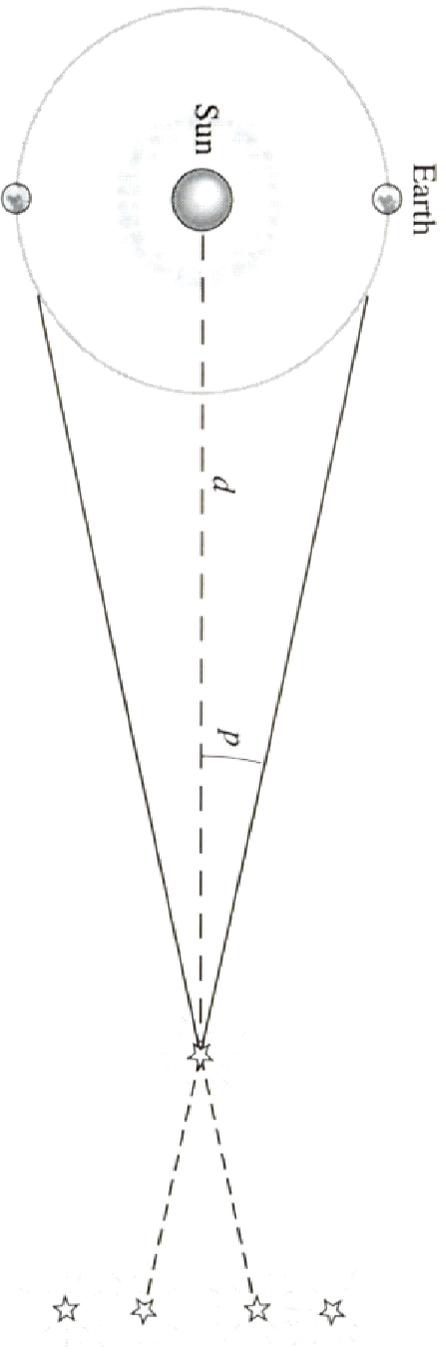
Stellar parallax: $d = 1/p''$ pc.

Precession and Nutation

Precession = movement of the direction of the Earth's rotational axis, caused mainly by Sun and Moon.

period: 25,764 years
magnitude: ~50"
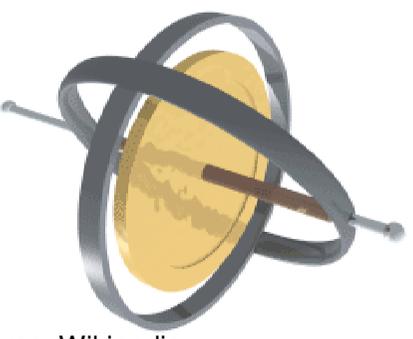
Nutation = tidal forces of Sun and Moon are continuously changing with their relative positions → cause precession to vary over time.

period: 18.6 years (largest component)
magnitude: ~9.2"

P

R

N
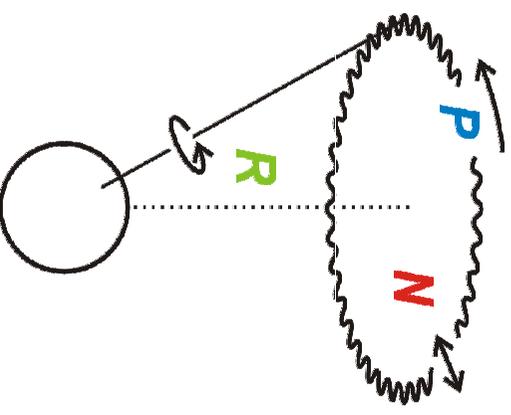
source: Wikipedia

- circular orbit, no axial tilt: analemma is a dot
- circular orbit and axial tilt: analemma is a figure of eight; N and S lobes equal
- eccentric orbit, no axial tilt: analemma is a straight E-W line along the equator

Photomontage illustrating an analemma pattern in the sky. Were one to take a photo of the sun each week for a year and combine the images taken, this pattern would be visible (from Wikipedia).

Ephemeris Time (ET)

The ephemeris time was defined by Simon Newcomb's (1898) tables* of the Earth's orbital motion around the Sun, later (more practically) by the orbital motion of the Moon around the Earth.

Definition:
One second (defined in 1900) = 1/31,556,925.9747 of the tropical year for 1900 January 0 at 12 hours ephemeris time. (since 1967 one uses atomic standards)

One Julian day has 86,400 seconds

One Julian century has 36,525 days

One tropical year has 365.2421879 days

*an ephemeris is a table of values that gives the positions of astronomical objects in the sky at a given time

Universal Time (UT)

Timescale based on the rotation of the Earth.

Reference location is Greenwich meridian :

UT = 12 hours + Greenwich hour angle

Ideally, noon Greenwich Mean Time is when the Sun crosses the Greenwich meridian.

But: variations in the Earth's orbit (nutation, ellipticity, axial tilt) cause differences of up to 16 minutes.

→ Use Newcomb's fictitious *mean* Sun – hence also called Greenwich mean time G.M.T.)

Nowadays, use atomic standards – won't slow down but require occasional one second discontinuities to keep the Earth rotation in phase with atomic clocks → UTC (coordinated universal time).

Sidereal Time (ST)

Sidereal time ≡ hour angle of the true vernal equinox

• The sidereal time is proportional to the UTC.

• It has the same value as the right ascension of any celestial body that is crossing the local meridian at that same moment.

• When one moves eastward 15° in longitude, the sidereal time increases by one hour.

Note: when ST = RA the object will be at culmination

When corrected for nutation → mean sidereal time.

Example: Local sidereal time (LST) at Leiden: 52°09'N 04°30'E:

Use, e.g. http://tycho.usno.navy.mil/sidereal.html to check

Sidereal Day

Every 24.00 hr the Sun reaches the highest position in the South
(= synodic day)

In the meantime, the Earth has rotated a bit more than 360 degrees as it orbits the Sun, (namely $360°/365.25 = 0.986°$).

Thus, a sidereal day is shorter:
$24hr \cdot 360°/360.986° = 23hr\ 56min$

distant star

Sun

12:00:00

Earth

11:56:04

12:00:00

23 h 56' 04"
a mean sidereal day

3' 56"

24 h
a solar day

source: Wikipedia