# Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

## 7th Lecture: 27 October 2010

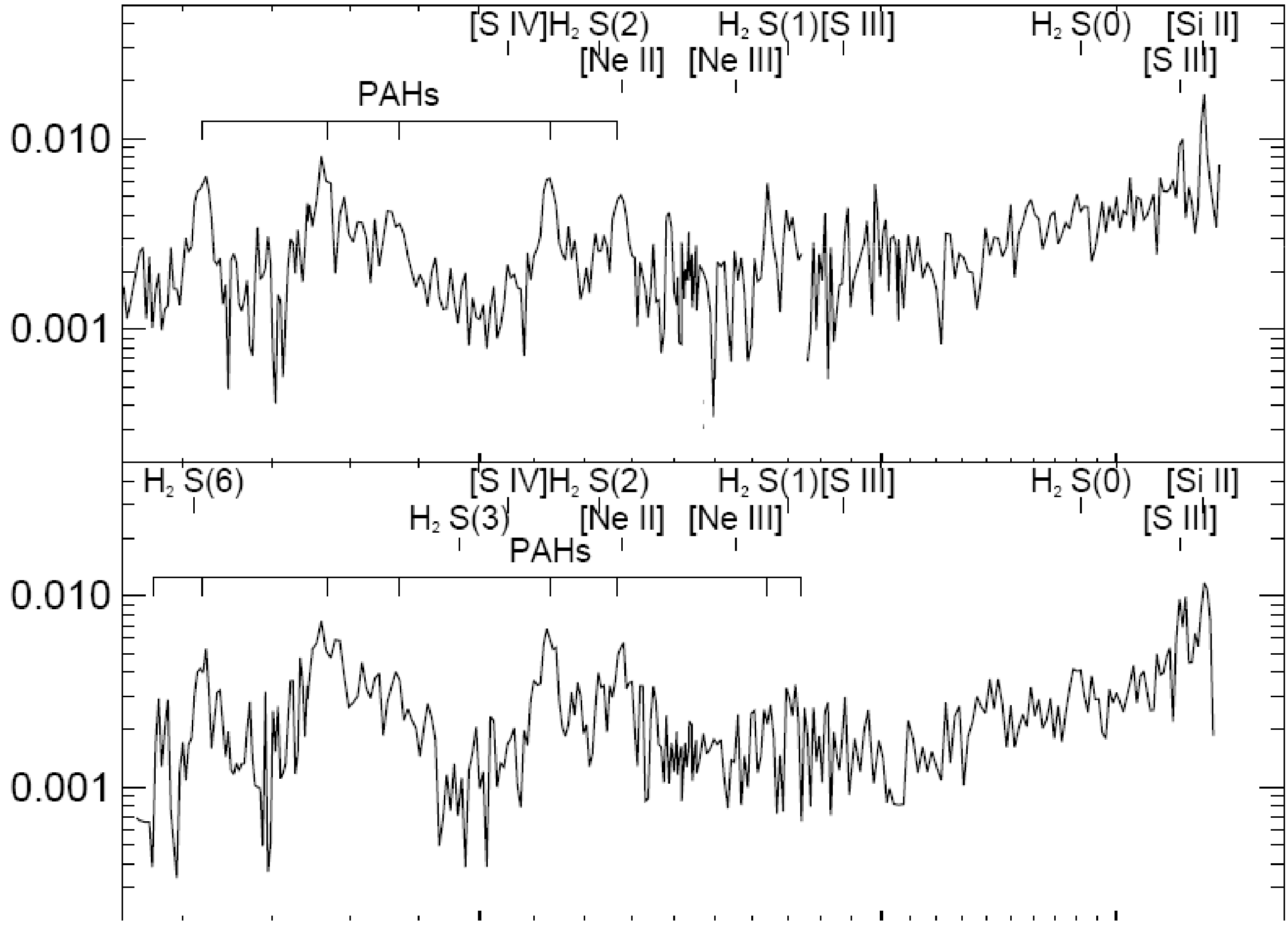
$$S_{cont} = \frac{\sigma h \lambda \sqrt{n_{pix}}\, 10^{30}}{SR\Delta\lambda A_{tel}\eta_D G \eta_{atm}\eta_{tot} t_{int}} \sqrt{\frac{2hc^2}{\lambda^5}\left(\frac{\varepsilon_T}{\exp\left[\frac{hc}{kT_T\lambda}\right]-1} + \frac{\varepsilon_A}{\exp\left[\frac{hc}{kT_A\lambda}\right]-1}\right)\eta_{tot}} \cdot$$

$$\cdot \sqrt{2\pi\left(1-\cos\left(\arctan\left(\frac{1}{2F\#}\right)\right)\right) D^2{}_{pix} \cdot \frac{\eta_D G \lambda}{hc} \cdot \Delta\lambda \cdot t_{int} + I_d t_{int} + N_{read}^2 n}$$

Sources: Lena book, Wikipedia, and many other sources

# Noise

# What is Noise here? And what is real?




# Noise Sources – some Examples

| Noise type | Source signal | Background flux |
|---|---|---|
| Photon shot noise | X | X |
| Scintillation | X | |
| Cosmic rays | | X |
| Thermal emission | | X |
| Strehl ratio (stability) | X | |
| Read noise | X | X |
| Dark current noise | X | X |
| CTE (CCDs) | X | X |
| Flat fielding (non-linearity) | X | X |
| Digitization noise | X | X |
| Other calibration errors | X | X |
| Image subtraction | X | |

# Example: Digitization Noise

Digitization = converting an analog signal into a digital signal using an Analog-to-Digital Converter (ADC).

The number of bits determines the dynamic range of the ADC. The resolution is $2^n$, where n is the number of bits.

Typical ADCs have:

12 bit: $2^{12}$ = 4096 quantization levels

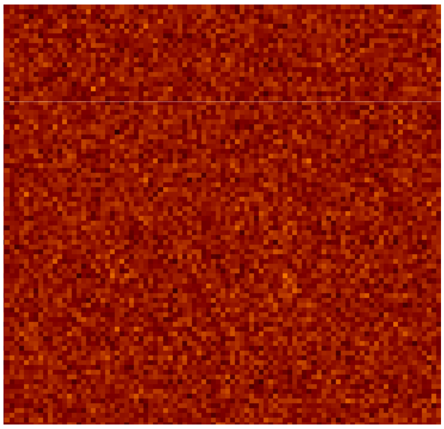
16 bit: $2^{16}$ = 65636 quantization levels

*Compare this to the detector pixel capacity (number of electrons)!*

# Noise Distribution: 1.Gaussian Noise

Gaussian noise is the noise following a Gaussian (normal) distribution.

It is often (incorrectly) called white noise, which refers to the (un-)correlation of the noise.

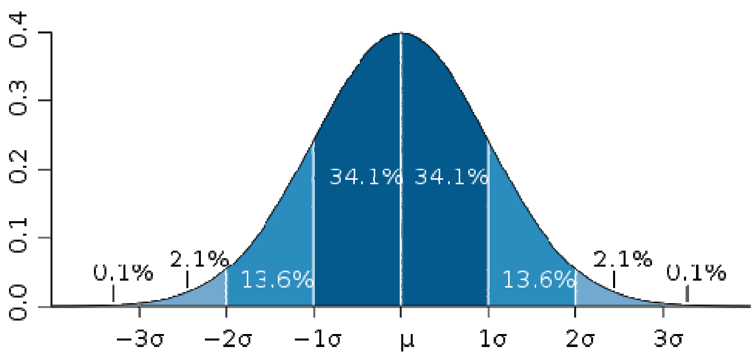


$$S = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$x$ is the actual value

$\mu$ is the mean of the distribution

$\sigma$ is the standard deviation of the distribution



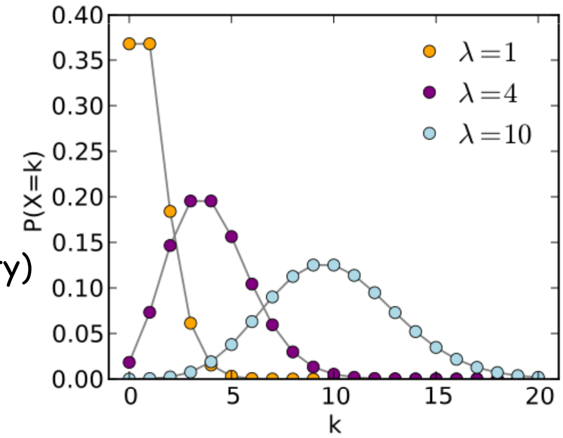

1-σ ~ 68%
2-σ ~ 95%
3-σ ~ 99.7%

# Noise Distribution: 2. Poisson Noise

Poisson noise is the noise following a Poissonian distribution.

It expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known *average rate* and *independently* of the time since the last event.

$$P(k,\lambda)=\frac{\lambda^k e^{-\lambda}}{k!}$$




$k$ is the number of occurrences of an event (probability)

$\lambda$ is the *expected* number of occurrences

- the mean (average) of $P(k,\lambda)$ is $\lambda$.
- the standard deviation of $P(k,\lambda)$ is $\sqrt{\lambda}$.

*Example: fluctuations in the detected photon flux between finite time intervals $\Delta t$. Detected are $k$ photons, while expected are on average $\lambda$ photons.*

# S/N Basics

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Signal = S;  Background = B;  Noise = N;  Telescope diameter = D

σ = Signal / Noise   ← measured as (S+B)-mean{B}
← total noise = $\sqrt{\sum (N_i)^2}$ (if statist. Independent)

Both S and N should be in units of events (photons, electrons, data numbers) per unit area (pixel, PSF size, arcsec$^2$).

Standard case: N = Poisson shot noise → √B

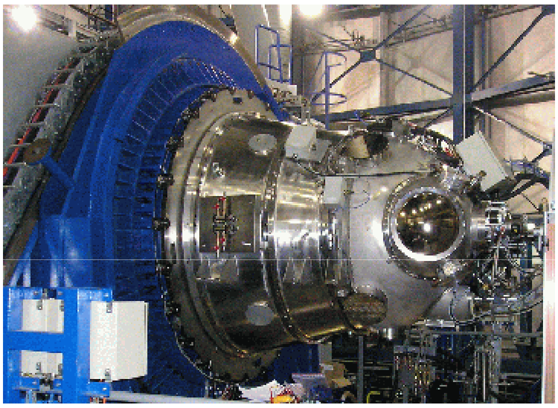
*Side note: noise between pixels is equivalent to successive measurements with one pixel – analogous  to throwing 5 dices versus one dice 5 times.*

Dependence on integration time $t_{int}$:

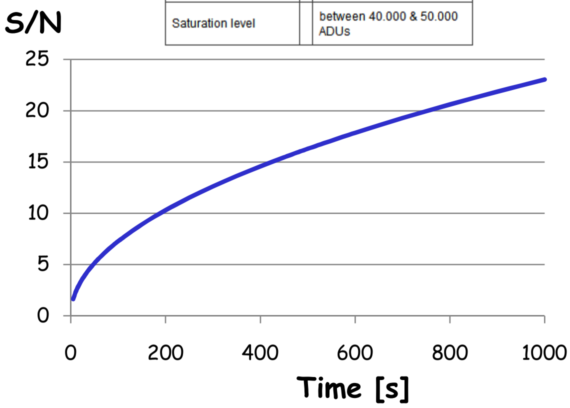
Consider integrating $n \times t_{int}$: $\sigma = \frac{n \cdot S}{\sqrt{n \cdot B}} = \sqrt{n}\,\frac{S}{N} \Rightarrow \boxed{\frac{S}{N} \propto \sqrt{t_{int}}}$

*You need to integrate four times as long to get twice the S/N*

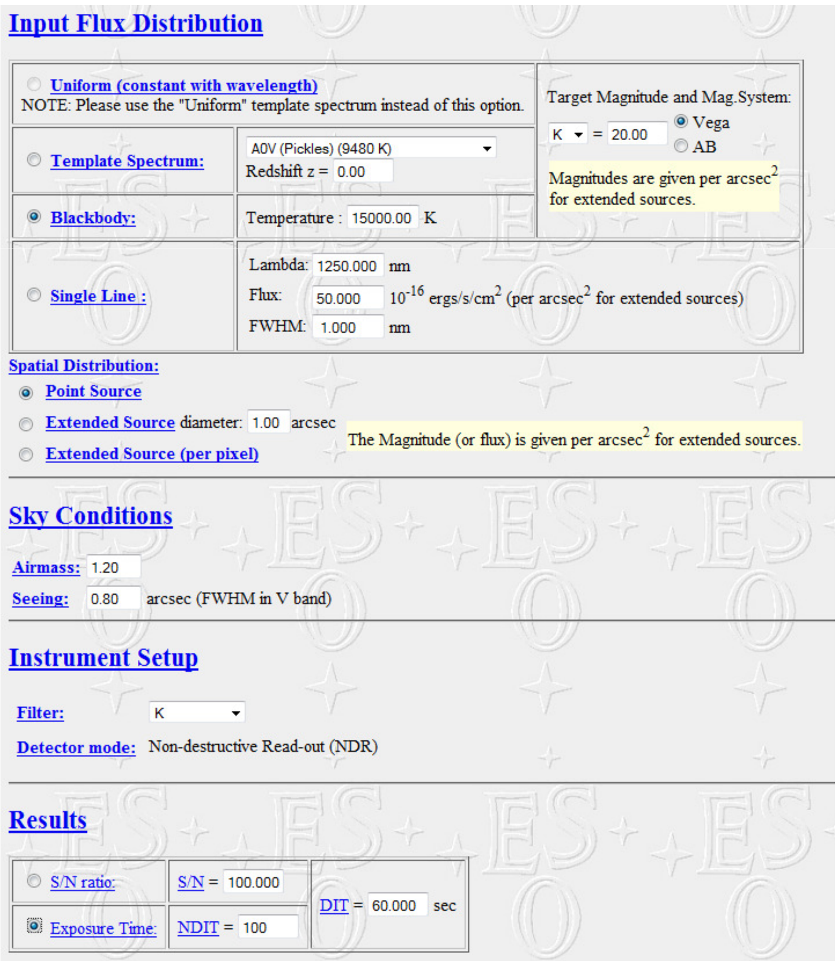
# Example: ESO's HAWK-I Wide Field Imager



| | |
|---|---|
| Operating temperature | 75K, controlled to 1mK |
| Dark current [e-/s] (at 75K) | between 0.10 & 0.15 |
| Read noise* (DCR) | ~ 12 e- |
| Read noise* (NDR) | ~ 5 e- |
| Linear range (1%) | 60.000e- (~30.000 ADUs) |
| Saturation level | between 40.000 & 50.000 ADUs |




http://www.eso.org/observing/etc/bin/ut4/hawki/script/hawkisimu



# Instrument Sensitivity

## Preface

The signal detection depends on the two main components:

1. the strength of the detected signal $S_{el}$
2. the total noise $N_{tot}$ of the system,

It can be characterized by the statistical significance of the detection σ (= signal-to-noise S/N)

$$\sigma = \frac{S_{el}}{N_{tot}}$$

Note:

(i) in this discussion we neglect quantum (shot) noise from the source.

(ii) we consider only point sources.

(iii) typically, the threshold for a "real" detection is taken as 3σ.

# Detected Signal

The detected signal $S_{el}$ depends on:

- the source flux density $S_{src}$ [photons $s^{-1}$ $cm^{-2}$ $\mu m^{-1}$]
- the integration time $t_{int}$ [s]
- the telescope aperture $A_{tel}$ [$m^2$]
- the transmission of the atmosphere $\eta_{atm}$
- the total throughput of the system $\eta_{tot}$, which includes:
  - the reflectivity of all telescope mirrors
  - the reflectivity (or transmission) of all instrument components, such as mirrors, lenses, filters, beam splitters, grating efficiencies, slit losses, etc.
- the Strehl ratio SR
- the detector responsivity $\eta_D G$
- the spectral bandwidth $\Delta\lambda$ [µm]

$$S_{el} = S_{src} \cdot SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{\mathrm{int}}$$

# Total Noise (1)

The total noise $N_{tot}$ depends on:

- the number of pixels $n_{pix}$ of one resolution element
- the total background noise per pixel $N_{back}$

$$N_{tot} = N_{back}\sqrt{n_{pix}}$$

where the total background noise $N_{back}$ depends on:

- the background flux density $S_{back}$
- the integration time $t_{int}$
- the detector dark current $I_d$
- the pixel read noise (N) and detector frames (n)

$$N_{back} = \sqrt{S_{back} \cdot t_{\mathrm{int}} + I_d \cdot t_{\mathrm{int}} + N_{read}^2 \cdot n}$$

# Total Noise (2)

The background flux density $S_{back}$ depends on:

- the total background intensity $B_{tot} = (B_T + B_A) \cdot \eta_{tot}$ where $B_T$ and $B_A$ are the thermal emissions from telescope and atmosphere, approximated by black body emission $B_{T,A} = \frac{2hc^2}{\lambda^5}\left[\frac{\varepsilon}{\exp\left[\frac{hc}{kT\lambda}\right]-1}\right]$
- the spectral bandwidth $\Delta\lambda$
- the pixel field of view $A \times \Omega = 2\pi\left(1-\cos\left(\arctan\left(\frac{1}{2F\#}\right)\right)\right)D^2{}_{pix}$
- the detector responsivity $\eta_D G$, and
- the photon energy $hc/\lambda$

$$S_{back} = B_{tot} \cdot A \times \Omega \cdot \frac{\eta_D G \lambda}{hc} \cdot \Delta\lambda$$

# Resulting Instrument Sensitivity

Putting it all together, the minimum detectable source signal is:

$$\sigma = \frac{S_{el}}{N_{tot}} = \frac{S_{src} \cdot SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{int}}{N_{back}\sqrt{n_{pix}}}$$

$$\Rightarrow S_{src} = \frac{\sigma \cdot \sqrt{S_{back} \cdot t_{int} + I_d \cdot t_{int} + N_{read}^2 \cdot n} \cdot \sqrt{n_{pix}}}{SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{int}}$$

Now we can calculate the unresolved line sensitivity $S_{line}$ [W/m$^2$] from the source flux $S_{src}$ [photons/s/cm$^2$/µm]:

$$S_{line} = \frac{hc}{\lambda} S_{src} \Delta\lambda \cdot 10^4$$

and with the relation $S_\lambda\left[\frac{W}{m^2 \mu m}\right] = S_\nu[Jy] \cdot 10^{-26}\frac{c}{\lambda^2}$

we can calculate the continuum sensitivity $S_{cont}$:

$$S_{cont} = \frac{hc}{\lambda} S_{src} \cdot 10^4 \cdot \frac{\lambda^2}{c} \cdot 10^{26} = 10^{30} h\lambda S_{src}$$

# S/N and Telescope Size

## Case 1: Seeing-limited "Point Source"

Signal = S; Background = B; Noise = N; Telescope diameter = D

$\theta_{seeing} \sim$ const

If detector Nyquist-sampled to $\theta_{seeing}$ :

$S \sim D^2$ (area)

$B \sim D^2 \rightarrow N \sim D$ (Poisson std.dev)

➔ $S/N \sim D$

➔ $t_{int} \sim D^{-2}$

## Case 2: Diffraction-limited extended Source

Signal = S; Background = B; Noise = N; Telescope diameter = D

"PSFØ" ~ const

If detector Nyquist sampled to $\theta_{diff}$: pixel ~ $D^{-2}$ but S ~ $D^2$

$D^2$ (telescope size) and $D^{-2}$ (pixel FOV) cancel each other
→ no change in signal

same for the background flux

➔ S/N ~ const → $t_{int}$ ~ const → *no gain for larger telescopes!*

Case 2B: offline re-sampling by a factor x (makes $\theta_{diff}$ x-times larger)

since $S/N \sim \sqrt{n_{pix}}$ → $S/N \sim \sqrt{x^2} = x$ → $t_{int} \sim x^{-2}$.

## Case 3: Diffraction-limited "Point Source"

Signal = S; Background = B; Noise = N; Telescope diameter = D

"$S/N = (S/N)_{light\ bucket} \cdot (S/N)_{pixel\ scale}$"

(i) Effect of telescope aperture:

S ~ $D^2$
B ~ $D^2$ → N ~ D
→ S/N ~ D

(ii) Effect of pixel FOV (if Nyquist sampled to $\theta_{diff}$):

S ~ const (pixel samples PSF = all source flux)
B ~ $D^{-2}$ → N ~ $D^{-1}$ → S/N ~ D

(i) and (ii) combined S/N ~ $D^2$ → $t_{int}$ ~ $D^{-4}$

→ *huge gain: 1hr ELT = 3 months VLT*