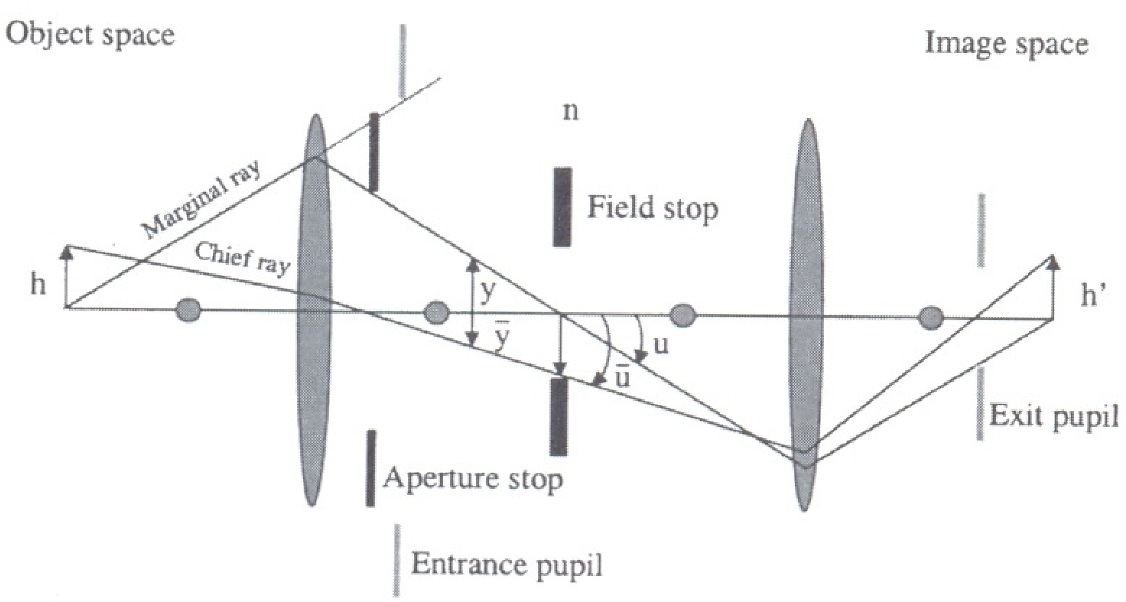
Astronomische Waarneemtechnieken
(Astronomical Observing Techniques)

6th Lecture: 20 October 2010



Based on "Astronomical Optics" by Daniel J. Schroeder, "Principles of Optics" by Max Born & Emil Wolf, the "Optical Engineer's Desk Reference" by William L. Wolfe, Lena book, and Wikipedia

Part I
Geometrical Optics

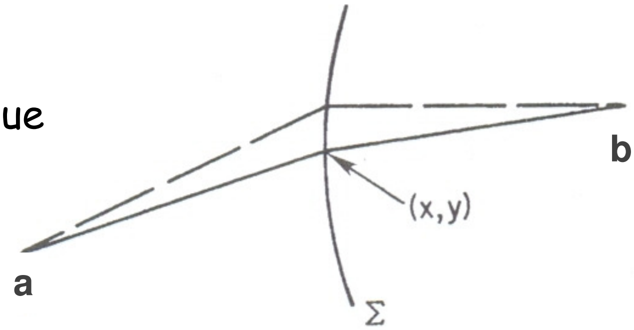
Part II
Diffraction Optics

Preface: Fermat's Principle

Consider two points, a and b, and various paths between them. The travel time between them is τ.

Condition: τ will have a stationary value

for the actual path: $\frac{\partial \tau}{\partial x} = \frac{\partial \tau}{\partial y} = 0$



Equivalently: travel time ⇔ optical path length (OPL)

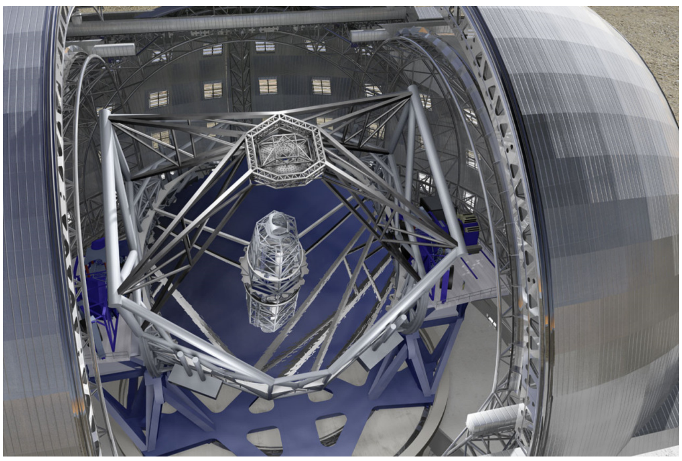
$$d(\mathrm{OPL}) = c\,dt = \frac{c}{\mathrm{v}} \cdot \mathrm{v}\,dt = n \cdot \frac{ds}{\mathrm{dt}}\,dt = n\,ds$$

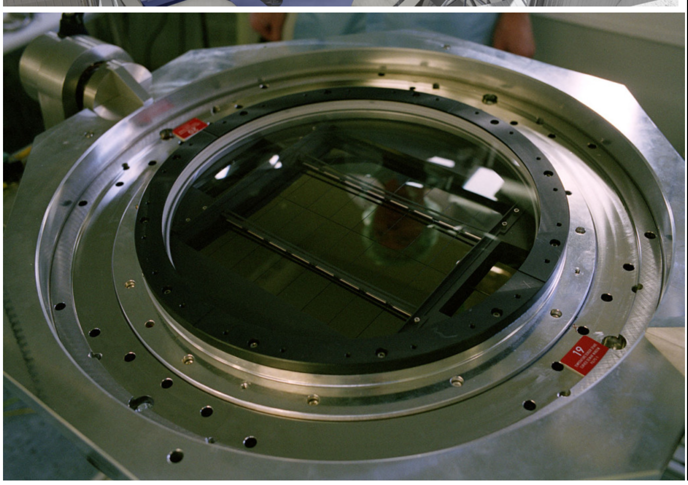
$$\mathrm{OPL} = \int_{\mathrm{a}}^{b} n\,ds$$

where v is the speed of light in the medium of index n.

Fermat's principle states that the OPL is the shortest distance a → b

Motivation

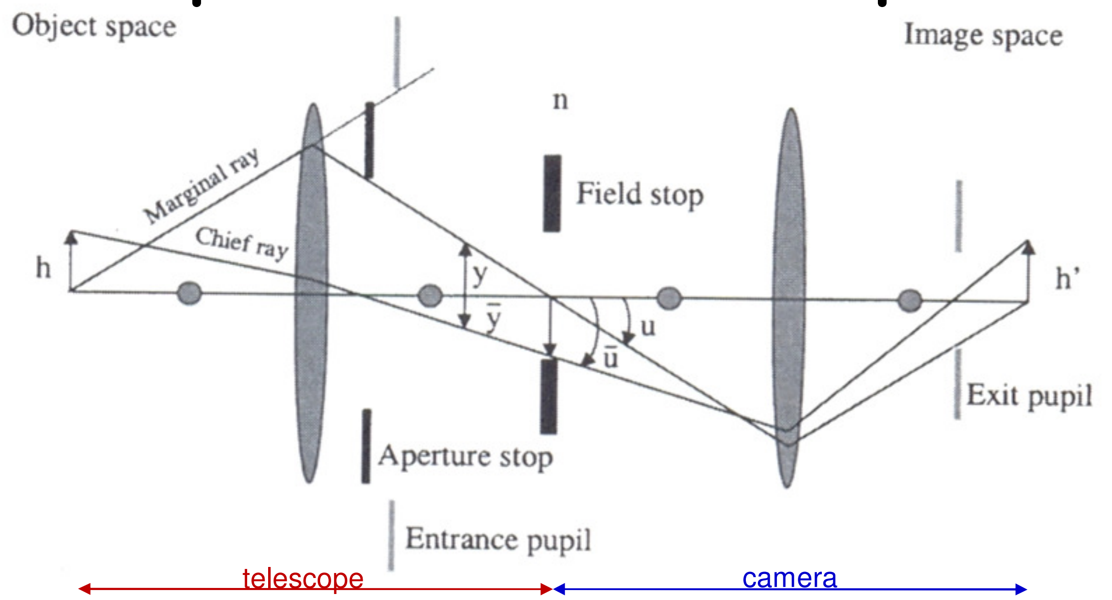




Understand how our tools work!

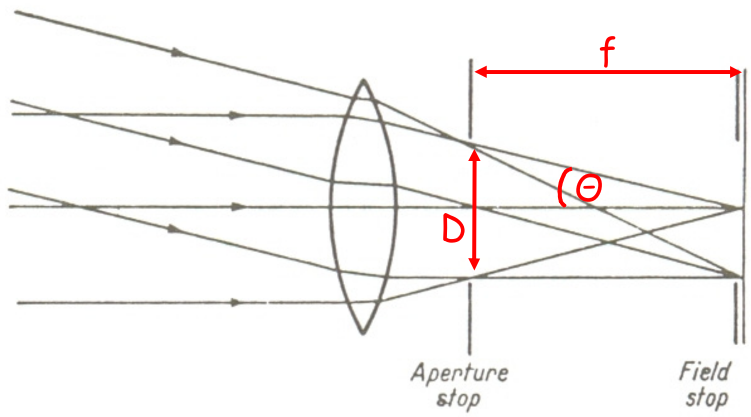
The ESO VLT UV-Visual Echelle Spectrograph

Aperture and Field Stops



Aperture stop:	determines the diameter of the light cone from an axial point on the object.
Field stop:	determines the field of view of the system.
Entrance pupil:	image of the aperture stop in the object space
Exit pupil:	image of the aperture stop in the image space
Marginal ray:	ray from an object point on the optical axis that passes at the edge of the entrance pupil
Chief ray:	ray from an object point at the edge of the field, passing through the center of the aperture stop.

The Speed of the System



The speed of an optical system is described by the numerical aperture NA and the *F* number, where:

$$\mathrm{NA} = n \cdot \sin\theta \qquad \text{and} \qquad F \equiv \frac{f}{D} = \frac{1}{2(\mathrm{NA})}$$

Generally, fast optics (large NA) has a high light power, is compact, has low tolerances and is difficult to manufacture. Slow optics (small NA) is just the opposite.

Aberrations

Generally, aberrations are departures of the performance of an optical system from the predictions of paraxial optics.

Until photographic plates became available, only *on-axis* telescope performance was relevant.

There are two categories of aberrations:

1. On-axis aberrations (defocus, spherical aberration)
2. Off-axis aberrations:
   a) Aberrations that degrade the image: coma, astigmatism
   b) Aberrations that alter the image position: distortion, field curvature

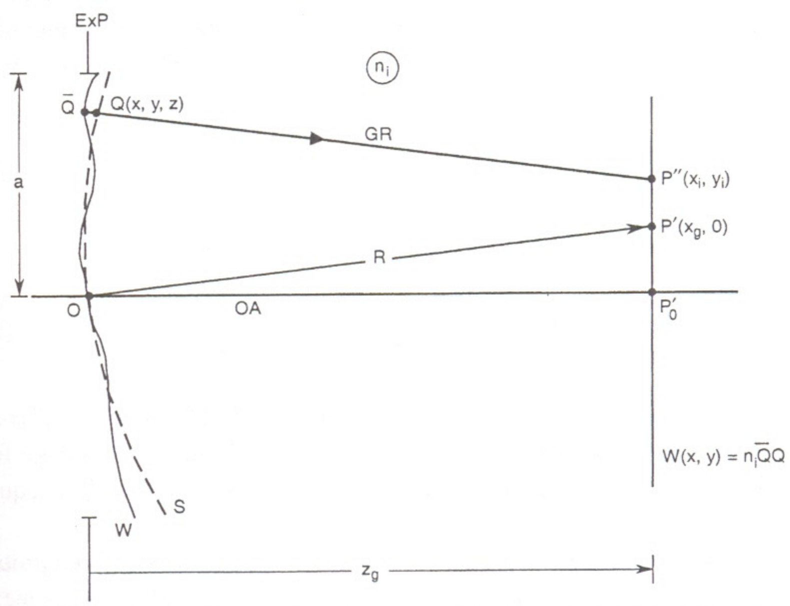
Relation between Wave and Ray Aberrations

Consider a reference sphere S of curvature R for a off-axis point P' and an aberrated wavefront W.

An "aberrated" ray from the object intersects the image plane at P".

The ray aberration is P'P".

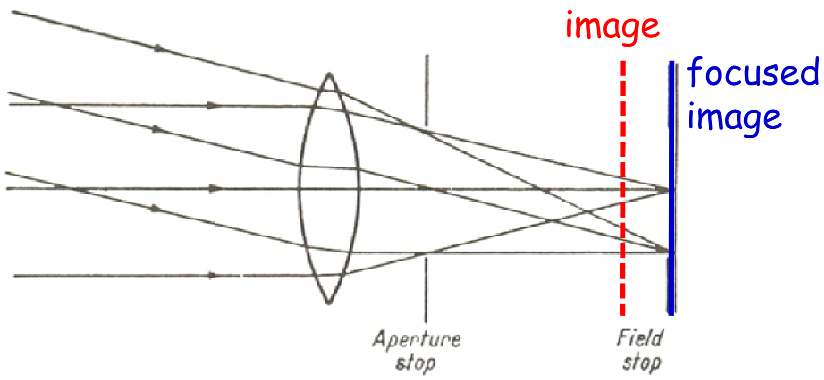
The wave aberration is $n \cdot \bar{Q}Q$



For small FOVs and a radially symmetric aberrated wavefront W(r) we can approximate the intersection with the image plane: $r_i = \frac{R}{n_i}\frac{\partial W(r)}{\partial r}$

Defocus

Defocus means "out of focus".



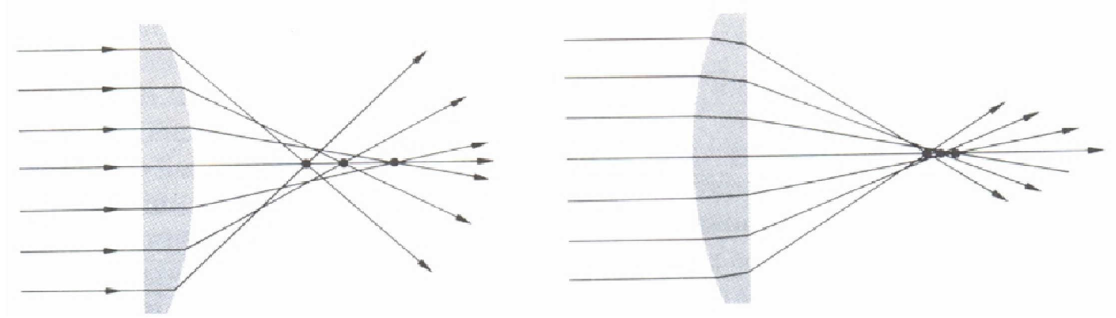
The amount of defocus can be characterized by the depth of focus:

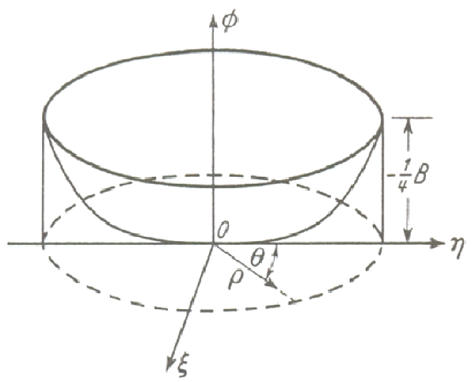
$$\delta = 2\lambda F^2 = \frac{\lambda}{2}\left(\frac{1}{\mathrm{NA}}\right)^2$$

The depth of focus usually refers to an optical path difference of λ/4.

Spherical Aberration

Rays further from the optical axis have a different focal point than rays closer to the optical axis:



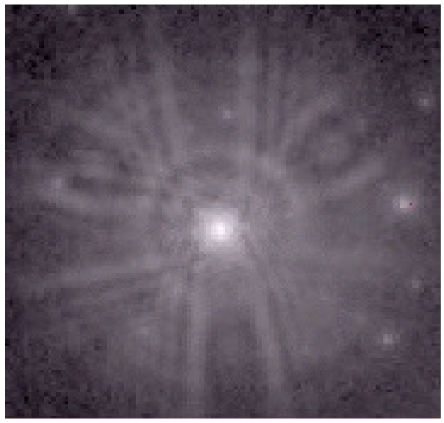


Spherical aberration

$\phi = -\frac{1}{4}B\rho^4$

φ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
ξ = ρ sinΘ; η = ρ cosΘ

Side note: the HST mirror

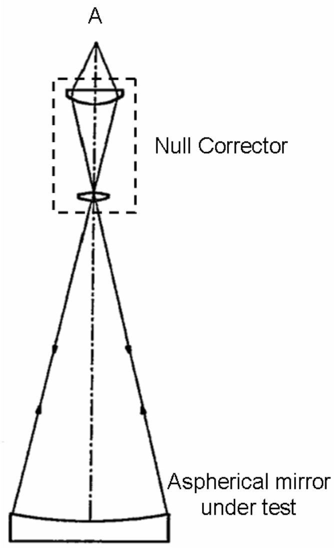


Optical problem: HST primary mirror suffers from spherical aberration.

Reason: the *null corrector* used to measure the mirror shape had been incorrectly assembled (one lens was misplaced by 1.3 mm).

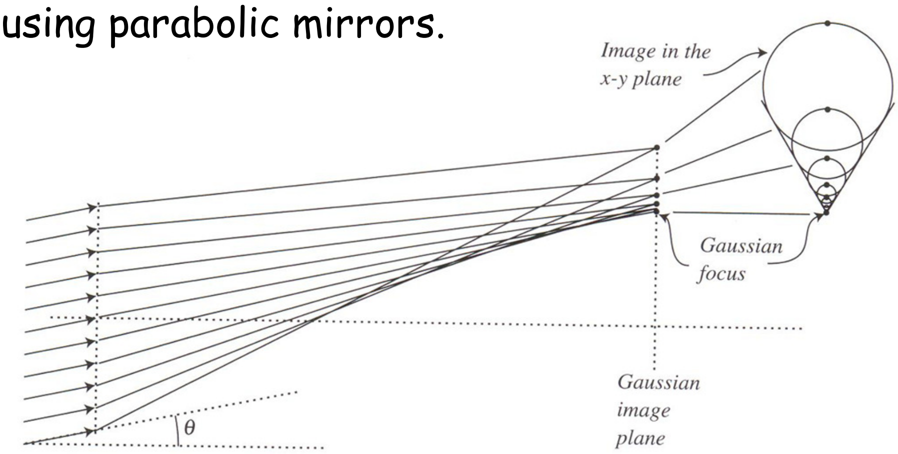
Management problem: The mirror manufacturer had analyzed its surface with other null correctors, which indicated the problem, but the test results were ignored because they were believed to be less accurate.

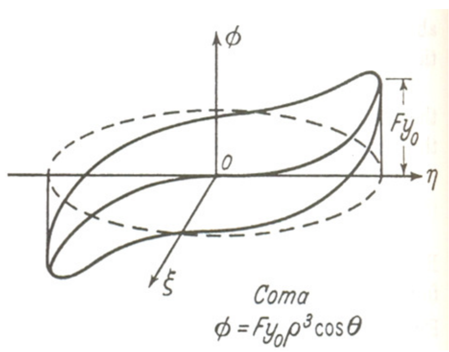
A null corrector cancels the non-spherical portion of an aspheric mirror figure. When the correct mirror is viewed from point A the combination looks precisely spherical.



Coma

Coma appears as a variation in magnification across the entrance pupil. Point sources will show a cometary tail. Coma is an inherent property of telescopes using parabolic mirrors.

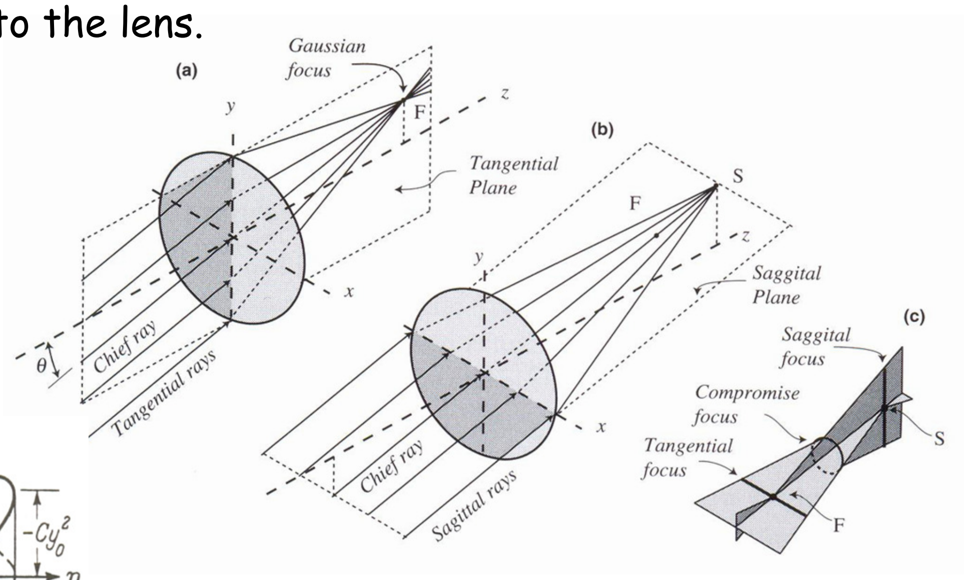


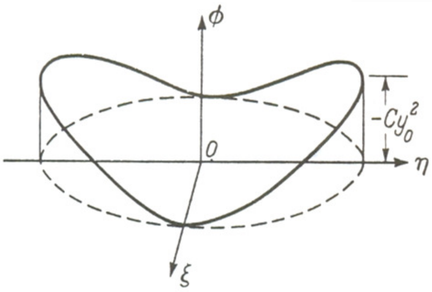


ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
y_0 = position of the object in the field

Astigmatism

Consider an off-axis point A. The lens does not appear symmetrical from A but shortened in the plane of incidence, the tangential plane. The emergent wave will have a smaller radius of curvature for the tangential plane than for the plane normal to it (sagittal plane) and form an image closer to the lens.





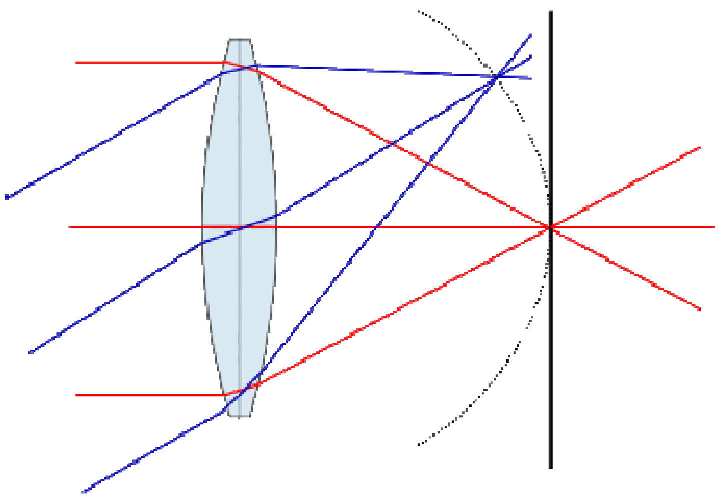
Astigmatism

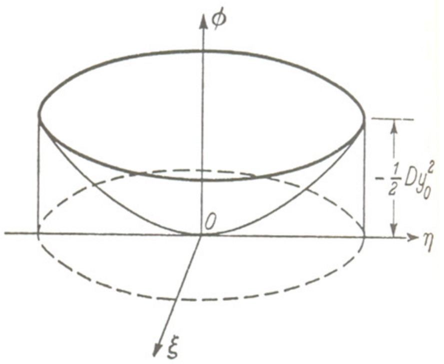
$\phi = -Cy_0^2\rho^2\cos^2\theta$

ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
y_0 = position of the object in the field

Field Curvature

Only objects close to the optical axis will be in focus on a flat image plane. Close-to-axis and far off-axis objects will have different focal points due to the OPL difference.





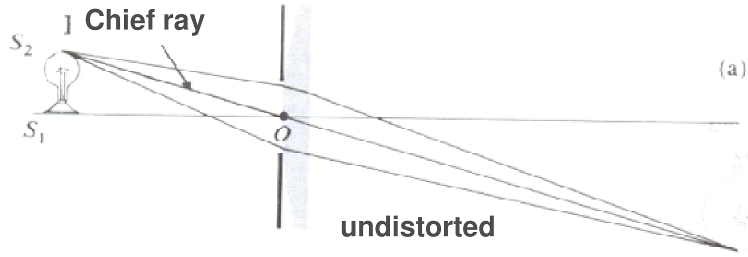
Curvature of field

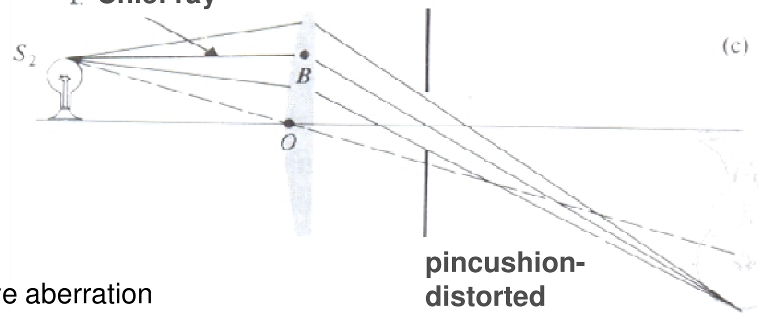
$\phi = -\frac{1}{2}Dy_0^2\rho^2$

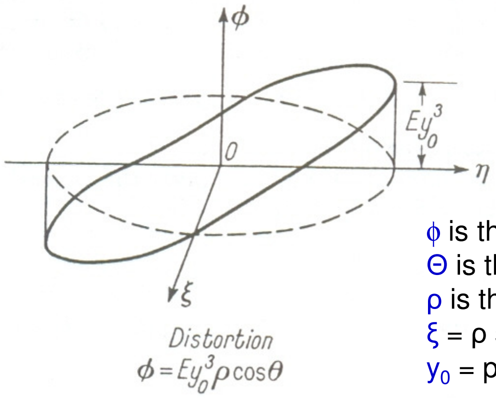
ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
y_0 = position of the object in the field

Distortion (1)

Straight lines on the sky become curved lines in the focal plane. The transversal magnification depends on the distance from the optical axis.





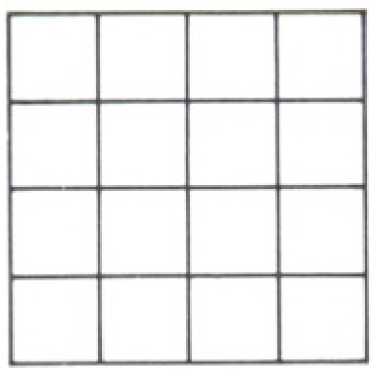
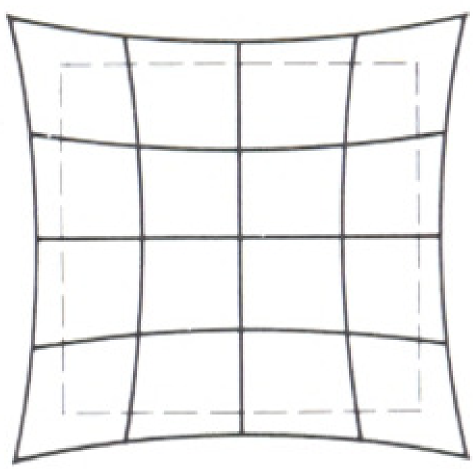
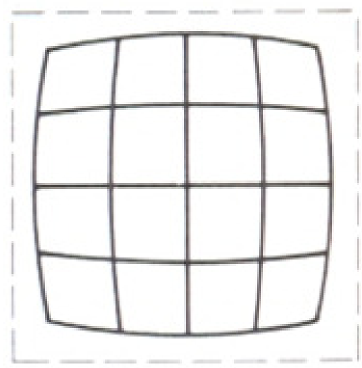


ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
y_0 = position of the object in the field

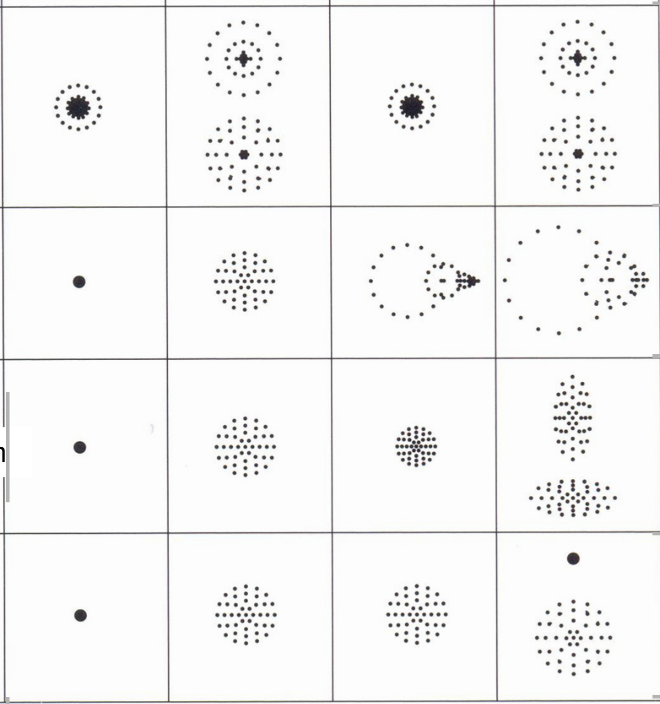
Distortion (2)

Generally there are two cases:

1. Outer parts have smaller magnification → barrel distortion
2. Outer parts have larger magnification → pincushion distortion

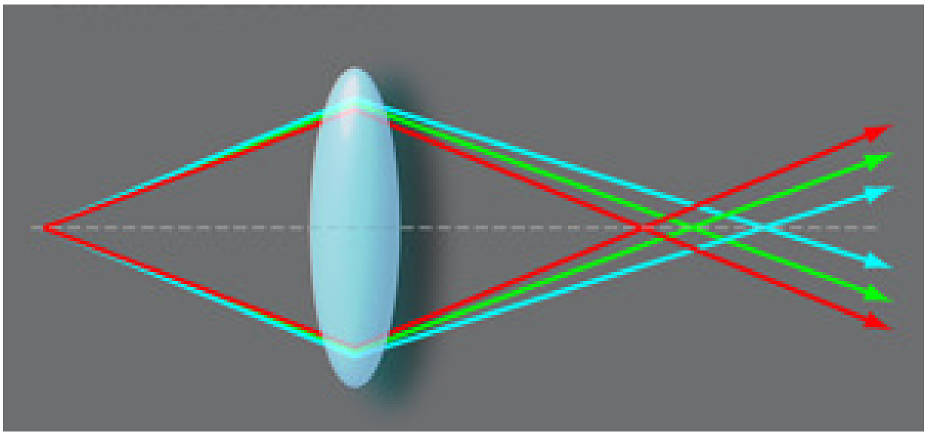
Summary: Primary Wave Aberrations



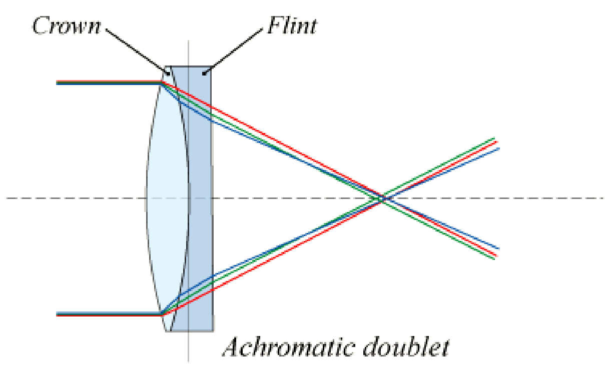
	On-axis focus	On-axis defocus	Off-axis	Off-axis defocus	Dependence on pupil size	Dependence on image size
Spherical aberration					$\sim\rho^4$	const.
Coma					$\sim\rho^3$	$\sim y$
Astigmatism					$\sim\rho^2$	$\sim y^2$
Field curvature					$\sim\rho^2$	$\sim y^2$
Distortion					$\sim\rho$	$\sim y^3$
Defocus					$\sim\rho^2$	const.

Chromatic Aberration

Since the refractive index n = f(λ), the focal length of a lens = f(λ) and different wavelengths have different foci. (Mirrors are usually achromatic).



Mitigation: use two lenses of different material with different dispersion → achromatic doublet



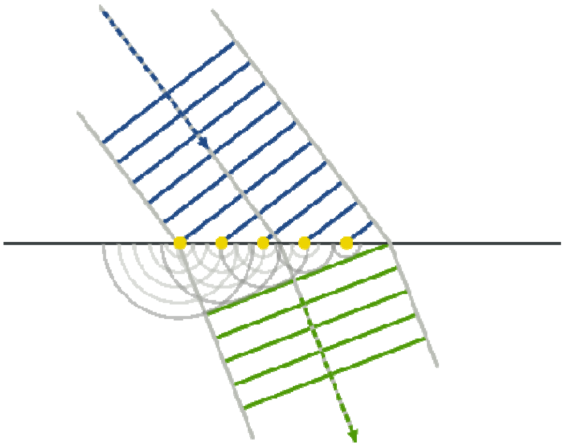
Part I
Geometrical Optics

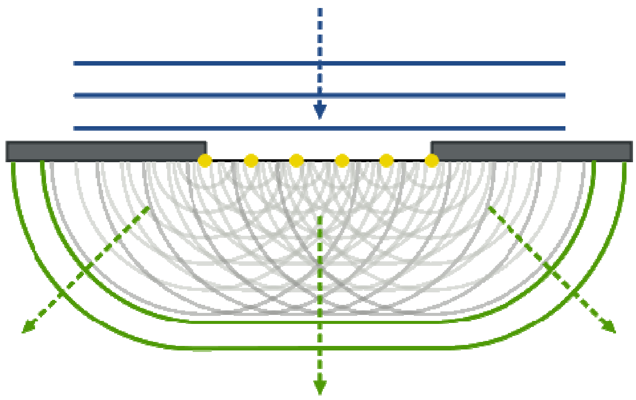
Part II
Diffraction Optics

Huygens-Fresnel Principle

Fermat's view: *"A wavefront is a surface on which every point has the same OPD."*

Huygens' view: *"At a given time, each point on primary wavefront acts as a source of secondary spherical wavelets. These propagate with the same speed and frequency as the primary wave."*





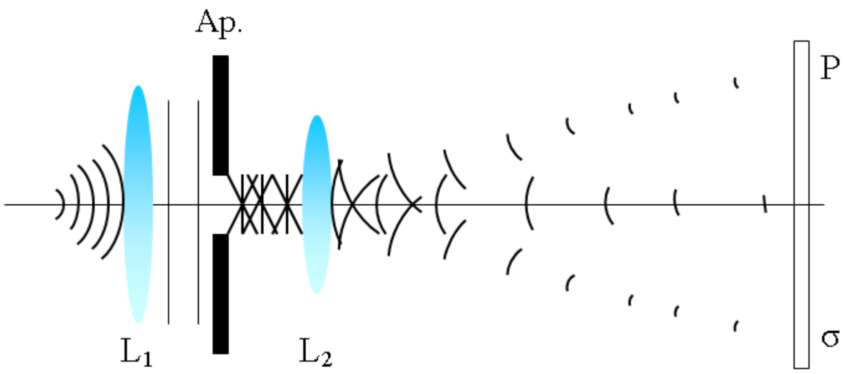
The Huygens-Fresnel principle was theoretically demonstrated by Kirchhoff (→ Fresnel-Kirchhoff diffraction integral)

Fresnel and Fraunhofer Diffraction

Fresnel diffraction = near-field diffraction

When a wave passes through an aperture and diffracts in the near field it causes the observed diffraction pattern to differ in size and shape for different distances.

For Fraunhofer diffraction at infinity (far-field) the wave becomes planar.



An example of an optical setup that displays Fresnel diffraction occurring in the ***near-field****. On this diagram, a wave is diffracted and observed at point σ. As this point is moved further back, beyond the Fresnel threshold or in the* ***far-field****, Fraunhofer diffraction occurs.*

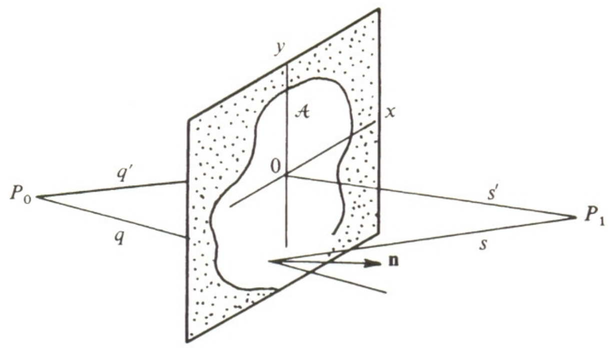
Fresnel: $F = \frac{r^2}{d \cdot \lambda} \geq 1$

Fraunhofer: $F = \frac{r^2}{d \cdot \lambda} << 1$

(where F = Fresnel number, r = aperture size and d = distance to screen).

Fraunhofer Diffraction at a Pupil

Consider a circular pupil function G(r) of unity within A and zero outside.



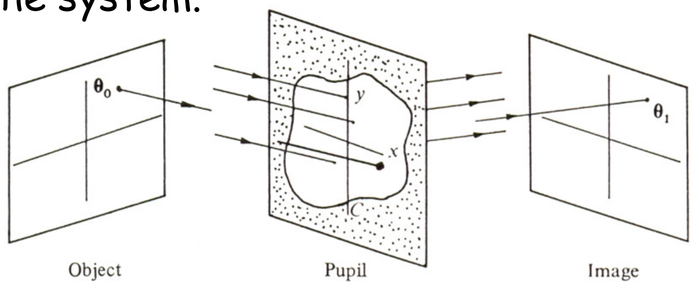
Theorem: *When a screen is illuminated by a source at infinity, the amplitude of the field diffracted in any direction is the Fourier transform of the pupil function characterizing the screen A.*

Mathematically, the amplitude of the diffracted field can be expressed as (see Lena book pp. 120ff for details):

$$V_1(\theta_1, t) = \lambda \sqrt{\frac{E}{A}} \iint_{screenA} G\left(\frac{r}{\lambda}\right) e^{-i2\pi(\theta_1 - \theta_0)\cdot\frac{r}{\lambda}} \frac{dr}{\lambda^2}$$

Relation between Object and Image

Let $V(\Theta_0)$ and $V(\Theta_1)$ be the complex field amplitutes of points in the object and image plane, respectively, and let $K(\Theta_0;\Theta_1)$ be the "transmission" of the system.



Then the image of an extended object can be described by:

$$V(\theta_1) = \iint_{object} V_0(\theta_0)\, K(\theta_1 - \theta_0)\, d\theta_0 \quad \text{where} \quad K(\theta) = \iint G(r)\, e^{-i2\pi\theta\frac{r}{\lambda}} \frac{dr}{\lambda^2}$$

And in Fourier space (Lena book p.123):

$$FT\{V(\theta_1)\} = FT\{V_0(\theta_0)\} \cdot FT\{K(\theta_0)\} = FT\{V_0(\theta_0)\} \cdot G(r)$$

In other words, the Fourier transform of the image equals the product of Fourier transform of the object and the pupil function G, which acts as a linear spatial filter.

Coherence Étendue

Not all sources provide coherent beams – consider a source of finite size, which subtends a solid angle Ω at a point of the screen.

Some area $S = \pi\rho^2$ of the screen corresponds to a beam étendue ε of:

$$\varepsilon = S\,\Omega = \pi\rho^2\, \pi\theta_0^2 \overset{u \equiv 2\pi\theta_o\rho/\lambda}{=} \frac{\lambda^2}{4} u^2$$

If we choose u=2 so that $2 = 2\pi\theta_o\rho/\lambda \;\Rightarrow\; \rho = \frac{\lambda}{\pi\theta_o}$ we get a degree of coherence greater than 50%:

$$\left|\gamma\left(\rho = \frac{\lambda}{\pi\theta_o}\right)\right| = \ldots = \frac{2|J_1(2)|}{2} = 0.577$$

(see Lena book p. 119 or details).

In other words, the beam remains coherent for an étendue of

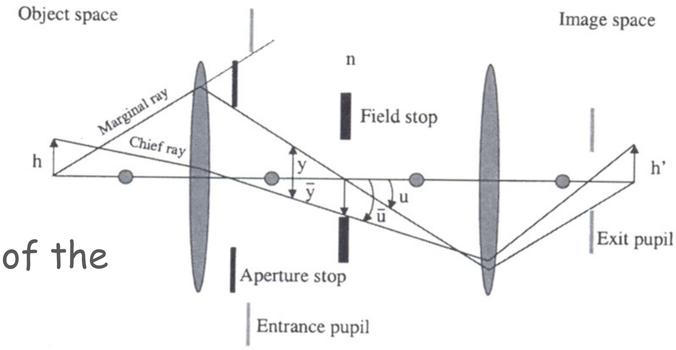
$$\varepsilon = \lambda^2$$

Étendue, AxΩ, and Throughput

The geometrical étendue (frz. `extent') is the product of area A of the source times the solid angle Ω (of the system's entrance pupil as seen from the source).
The geometric étendue may be viewed as the maximum beam size the instrument can accept.

Hence, the étendue is also called acceptance, throughput, or A×Ω product.

The étendue never increases in any optical system. A perfect optical system produces an image with the same étendue as the source.



Here $A=h^2\pi$, and Ω is given by the angle of the marginal ray.

Point Spread Function (1)

All physical pupils have finite sizes → cut-off frequencies $\omega_c = \left(u_c^2 + v_c^2\right)^{1/2}$ must exist. The pupil function G(r) acts as a low-pass filter on the spatial frequencies, equivalent to a convolution:

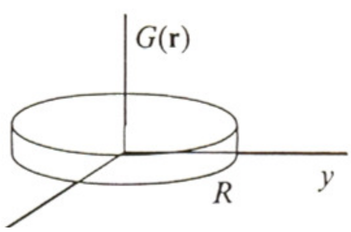
$$FT\{V(\theta_1)\} = FT\{V_0(\theta_0)\}\cdot FT\{K(\theta_0)\} \Leftrightarrow I(\theta_1) = I(\theta_0) * PSF(\theta)$$

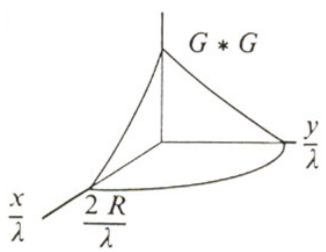
According to the Nyquist-Shannon sampling theorem I(Θ) shall be sampled with a rate of at least $\Delta\theta = 1/2\omega_c$.

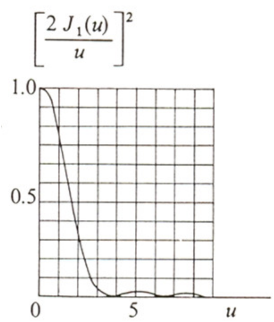
Now consider circular telescope aperture with: $G(r) = \Pi\left(\frac{r}{2r_0}\right)$

Graphically:

pupil function G(r) → autocorrelation G(r)*G(r) → and its MTF



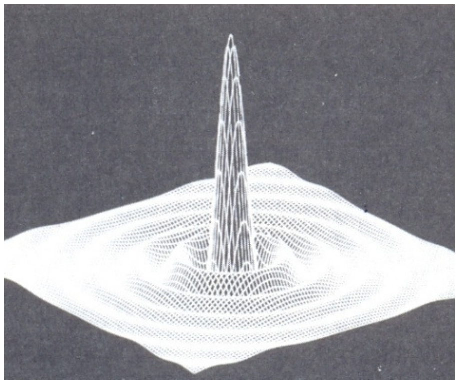




Point Spread Function (2)

When the circular pupil is illuminated by a point source $[I_0(\Theta) = \delta(\Theta)]$ then the resulting PSF is described by a 1st order Bessel function:

$$I_1(\theta) = \left(\frac{2J_1(2\pi\, r_0 \theta / \lambda)}{2\pi\, r_0 \theta / \lambda} \right)^2$$

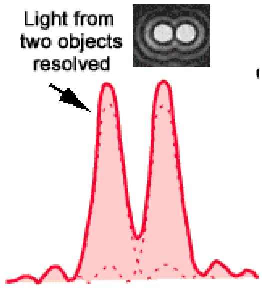


This is also called the Airy function.
The radius of the first dark ring (minimum) is at:

$$r_1 = 1.22 \lambda F \qquad \text{or} \qquad \alpha_1 = \frac{r_1}{f} = 1.22 \frac{\lambda}{D}$$

The PSF is often characterized by the half power beam width (HPBW) in angular units, to indicate the angular resolution of the image.

Remember: the Rayleigh criterion states that two sources can be resolved if the peak of the second source is no closer than the 1st dark Airy ring of the first source.



Point Spread Function (3)

Most "real" telescopes have a central obscuration, which modifies our simplistic pupil function $G(r) = \Pi(r / 2r_0)$

The resulting PSF can be described by a modified function:

$$I_1(\theta) = \frac{1}{(1-\varepsilon^2)^2} \left(\frac{2J_1(2\pi\, r_0 \theta / \lambda)}{2\pi\, r_0 \theta / \lambda} - \varepsilon^2 \frac{2J_1(2\pi\, r_0 \varepsilon \theta / \lambda)}{2\pi\, r_0 \varepsilon \theta / \lambda} \right)^2$$

where **ε** is the fraction of central obscuration to total pupil area.

Radii of Dark Rings in Airy Pattern[a,b]

ε	w_1	w_2	w_3
0.00	1.220	2.233	3.238
0.10	1.205	2.269	3.182
0.20	1.167	2.357	3.087
0.33	1.098	2.424	3.137
0.40	1.058	2.388	3.300
0.50	1.000	2.286	3.491
0.60	0.947	2.170	3.389

[a] Subscript on w is the number of the dark ring starting at the innermost ring.
[b] $w = v/\pi$.

Astronomical instruments sometimes use a phase mask to reduce the secondary lobes of the PSF (from diffraction at "hard edges"). Phase masks introduce a position dependent phase change. This is called apodisation.

Strehl Ratio

A convenient measure to assess the quality of an optical system is the Strehl ratio.

The Strehl ratio (SR) is the ratio of the observed *peak intensity* of the PSF compared to the theoretical maximum peak intensity of a point source seen with a perfect imaging system working at the diffraction limit.

Using the wave number $k=2\pi/\lambda$ and the RMS wavefront error ω one can calculate that:

$$SR = e^{-k^2\omega^2} \approx 1 - k^2\omega^2$$

Examples:

- A SR > 80% is considered diffraction-limited → average WFE ~ $\lambda/14$
- A typical adaptive optics system delivers SR ~ 10-50% (depends on λ)
- A seeing-limited PSF on an 8m telescope has a SR ~ 0.1-0.01%.

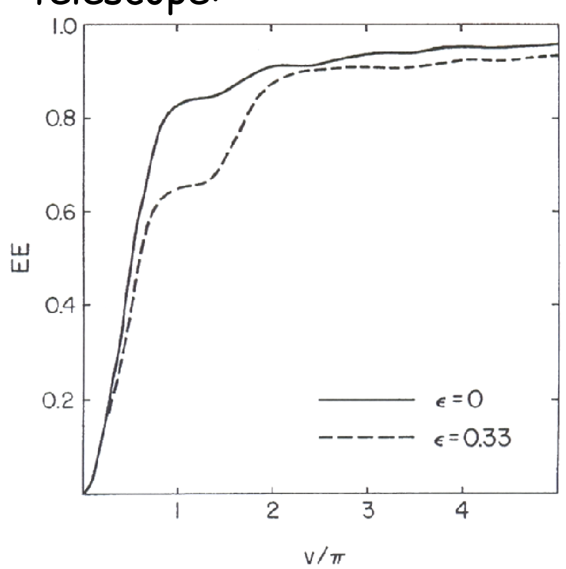
Encircled Energy

Q: What is the maximum concentration of light within a small area?
The fraction of the total PSF intensity within a certain radius is given by the encircled energy (EE):

$$EE(r) = 1 - J_0^2\left(\frac{\pi r}{\lambda F}\right) - J_1^2\left(\frac{\pi r}{\lambda F}\right)$$

F is the f/# number

Note that the EE depends strongly on the central obscuration ε of the telescope:



Encircled Energy Fraction within Airy Dark Rings[a]

ε	EE_1	EE_2	EE_3
0.00	0.838	0.910	0.938
0.10	0.818	0.906	0.925
0.20	0.764	0.900	0.908
0.33	0.654	0.898	0.904
0.40	0.584	0.885	0.903
0.50	0.479	0.829	0.901
0.60	0.372	0.717	0.873

[a] Subscript on EE is number of dark ring starting at innermost ring.