Exercises Astronomical Observing Techniques, Set 12

Exercise 1

Express the Full Width Half Maximum (FWHM) of a Gaussian in σ.

Exercise 2

In a Fourier Transform Spectrometer, the electric fields of the interfering beams arriving at the detector, are represented by:

$$\mathbf{E_1} = \mathbf{E_{01}} \cos(kx_1 - \omega t) \tag{1}$$

$$\mathbf{E_2} = \mathbf{E_{02}} \cos(kx_2 - \omega t) \tag{2}$$

where the two beams have experienced a physical path difference of $x = x_2 - x_1$. Remember that $k = 2\pi/\lambda$, k is the wavenumber. The time averaged irradiance for the k component is then

$$I_k = < (\mathbf{E_1} + \mathbf{E_2})^2 > \tag{3}$$

a) Write out the terms of the quadratic. Remember that $\mathbf{E_1}$ and $\mathbf{E_2}$ are vectors.

b) Now rewrite the interference term to a single cosine. Use $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$ and $< \cos^2 \omega t > = < \sin^2 \omega t > = 1/2$ and $< \cos \omega t \sin \omega t > = 0$.

c) Proof that $I_k = 2I_0(1 + \cos\delta)$ for $\delta = kx$ and $\mathbf{E_{01}} = \mathbf{E_{02}}$. Use $I_0 = \frac{1}{2}E_{01}^2$.

d) The irradiance over all wavelengths I is given by $\int_0^\infty I(k)dk$, with $I(k) = I_k$. Proof that moving the mirror of the Fourier Transform Spectrometer will give you the interferogram $I(x)$, which is the Fourier Transform of the spectral distribution $I(k)$.