Exercises Astronomical Observing Techniques, Set 5

Exercise 1

The Fourier pairs $f(x)$ and $F(s)$ are defined as follows:
$\int_{-\infty}^{+\infty} f(x)e^{-2\pi ixs}\,dx = F(s)$: $\mathcal{F}\{f(x)\} = F(s)$, the Fourier transform of $f(x)$ and
$\int_{-\infty}^{+\infty} F(s)e^{2\pi ixs}\,ds = f(x)$: $\hat{\mathcal{F}}\{F(s)\} = f(x)$, the inverse Fourier transform of $F(s)$

a) show that: $\mathcal{F}\{a(f(x)) + b(g(x))\} = a\mathcal{F}\{f(x)\} + b\mathcal{F}\{g(x)\}$

b) show that: $\mathcal{F}\{f(x-a)\} = e^{-2\pi ias}F(s)$

c) show that: $\mathcal{F}\{f(ax)\} = \frac{1}{|a|}F(s/a)$

Exercise 2

Compute the Fourier transforms (definition in Exercise 1) of:

a) $\delta(x)$

b) $\delta(x+a)$

c) $e^{-x^2\pi}$

d) $\frac{1}{2}\{\delta(x+\frac{1}{2}) + \delta(x-\frac{1}{2})\}$

e) $\prod(x), 1 \text{ for } |x| < \frac{1}{2}a, \text{ else } 0$

Exercise 3

Show that $\mathcal{F}\{\frac{df(x)}{dx}\} = 2\pi isF(s)$

Exercise 4

A galaxy has a spectral irradiance (flux density) of 10 mJy at a wavelength of 0.55 μm. Calculate the spectral irradiance in units of W m^{-3} and W m^{-2} Hz^{-1}.