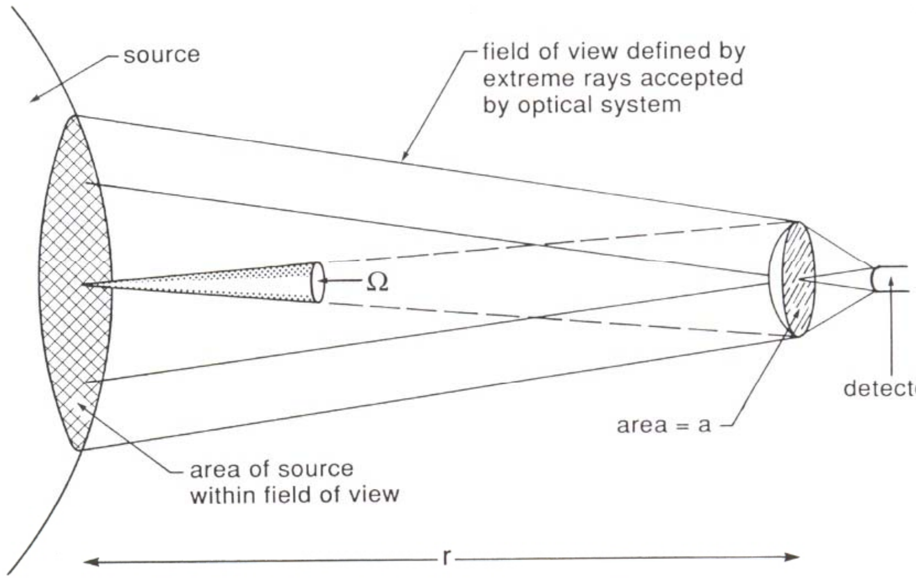
Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

2nd Lecture: 17 September 2008



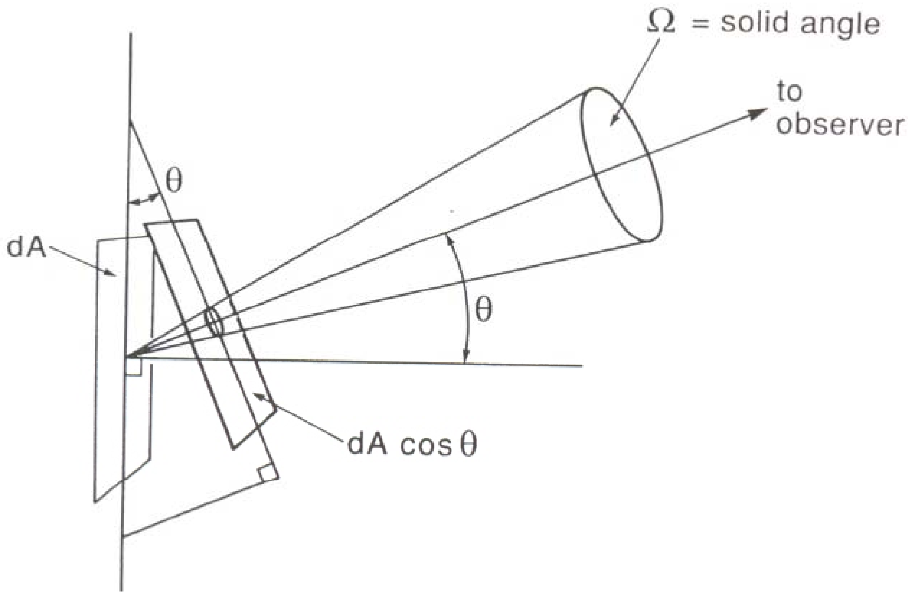
Based on "Observational Astrophysics" (Springer) by P. Lena, F. Lebrun & F. Mignard, 2nd edition – Chapter 3; and Rieke "Detection of Light"

1. Radiometry

Radiometry = the physical quantities associated with the energy transported by electromagnetic radiation.

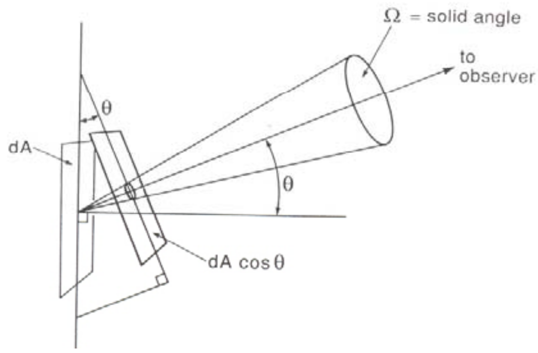
Photon energy: $E_{ph} = h\nu = \frac{hc}{\lambda}$

with h = Planck's constant [$6.626 \cdot 10^{-34}$ Js]



Emission of an Object

Consider a projected area of a surface element dA onto a plane perpendicular to the direction of observation.
θ is the angle between both planes.



The spectral radiance L_ν or specific intensity I_ν = the power *leaving* a unit projected area [m^2] into a unit solid angle [sr] and unit frequency interval [Hz].

It is measured in units of [$W\ m^{-2}\ sr^{-1}\ Hz^{-1}$] in frequency space.

The spectral radiance in wavelength space L_λ has units [$W\ m^{-3}\ sr^{-1}$]

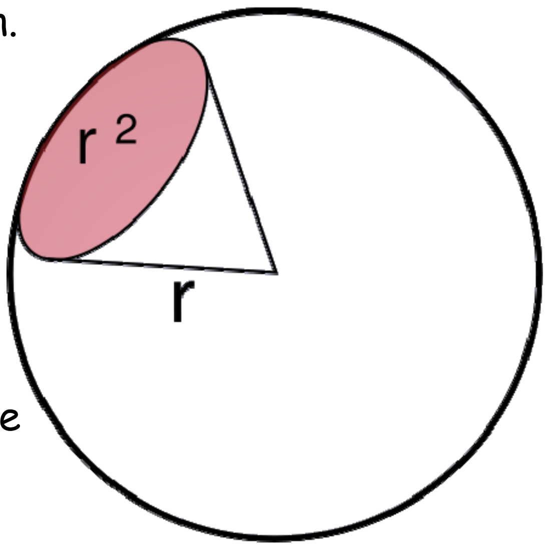
The radiance L or intensity I is the spectral radiance integrated over all frequencies or wavelengths. Units are [$W\ m^{-2}\ sr^{-1}$].

Side note: Steradian

Steradian [sr] = the dimensionless SI unit of the solid angle.

One steradian is the solid angle at the center of a sphere of radius r under which a surface area of area r^2 is seen.

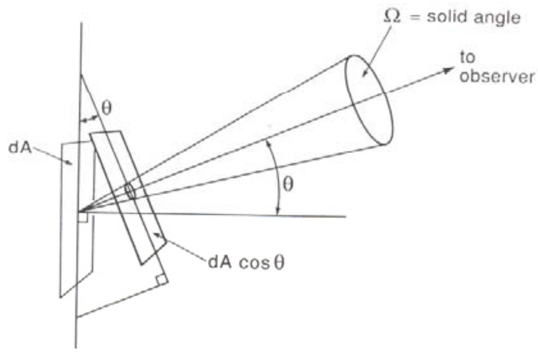
A complete sphere = 4π sr.
1 sr = $(180 deg/\pi)^2$ = 3282.80635 deg^2.



In two dimensions, the angle in radians is the arc length it cuts out:
θ = (arc length s) / (radius of the circle r)

In three dimensions, the solid angle in steradians is the area it cuts out:
Ω = (surface area S) / (radius of the sphere r)

Emission of an Object (2)



The radiant exitance M is the integral of the radiance over the solid angle Ω.

It measures the total power emitted per unit surface area. Units are [W m^{-2}].

For Lambertian sources we get:

$$M = \int L\cos\theta d\Omega = 2\pi L\int_0^{2\pi}\sin\theta\cos\theta d\theta = \pi L$$

The flux Φ or luminosity L emitted by the source is product of radiant exitance and total surface area of the source – hence the power emitted by the *entire* source.

For example, a source of radius R has:

$$\Phi = 4\pi R^2 M = 4\pi^2 R^2 L$$

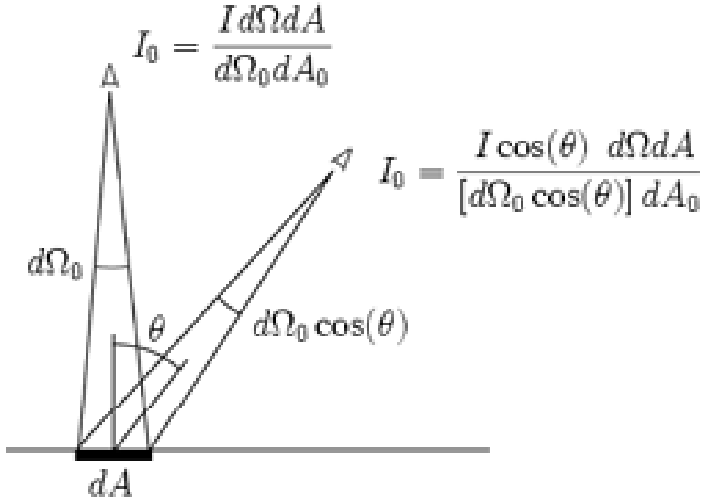
Side note: Lambertian Emitters

Perfect black bodies obey Lambert's law (1760):

$$dP_\nu = B_\nu dS\cos\theta\Omega d\nu = B_\nu d\sigma d\Omega d\nu$$

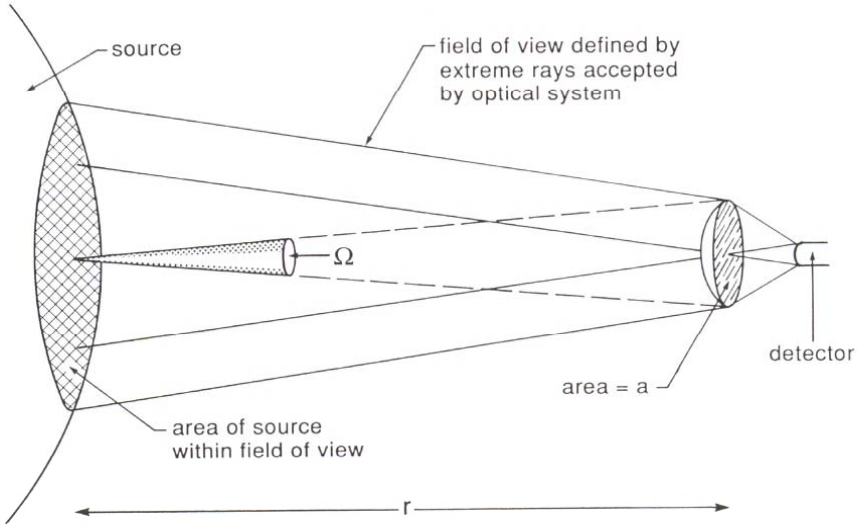
meaning that the intensity is independent of the direction θ of observation.

When a Lambertian surface is viewed from an angle θ then dΩ is decreased by cos(θ) but the size of the observed area A is increased by the corresponding amount. Example: the Sun is almost a perfect Lambertian radiator (except for the limb) with a uniform brightness across the disk (although only the center is on-axis).



Computing the received Power

A detector system usually accepts radiation only from a limited range of directions, the field of view (FOV).

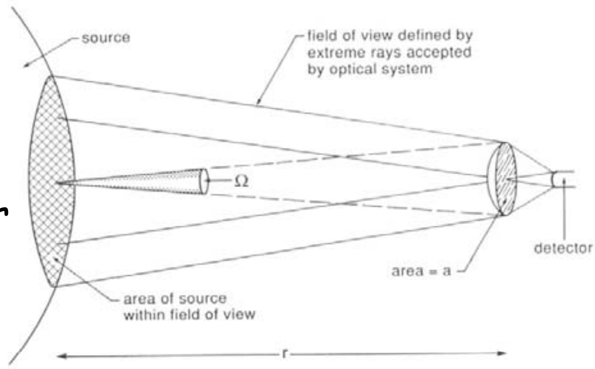


The relevant area of the source depends on FOV and distance r.

The detected power is then the radiance times the source area within the FOV times the solid angle subtended by the optical system as viewed from the source.

Computing the received Power (2)

Here we assume that the entire source of radius R (or area πR^2) lies within the FOV.



The solid angle subtended by the detector system is:

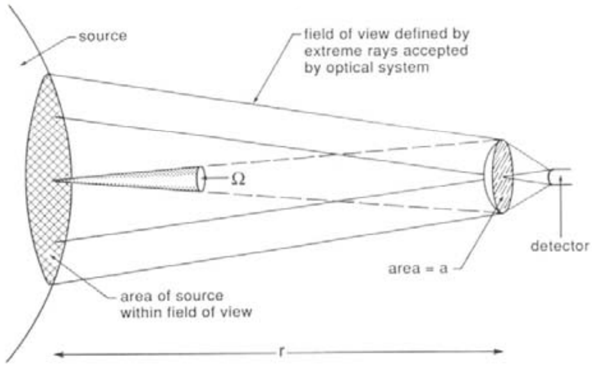
$$\Omega = \frac{a}{r^2}$$

where *a* is the area of the entrance aperture and *r* is the distance to the source.

For a circular aperture: $\Omega = 4\pi \sin^2\left(\frac{\theta}{2}\right)$

where θ is the half angle of the right cone.

Computing the received Power (3)



The irradiance E is the power received at a unit surface element from the source. Units are [W m^{-2}]. To compute E:

1. multiply M (=π·L) by surface area A of the source to get the flux Φ.
2. divide the flux Φ by the area of a sphere of radius r.

That yields: $$E = \frac{AL}{4r^2}$$

The spectral irradiance E_ν or flux density is the irradiance per frequency or wavelength interval: $$E_\nu = \frac{AL_\nu}{4r^2}$$

The units are [W m^{-2} Hz^{-1}] or [Jy].

Note: 1 Jansky = 10^{-26} W m^{-2} Hz^{-1}.

Overview of Radiometric Quantities

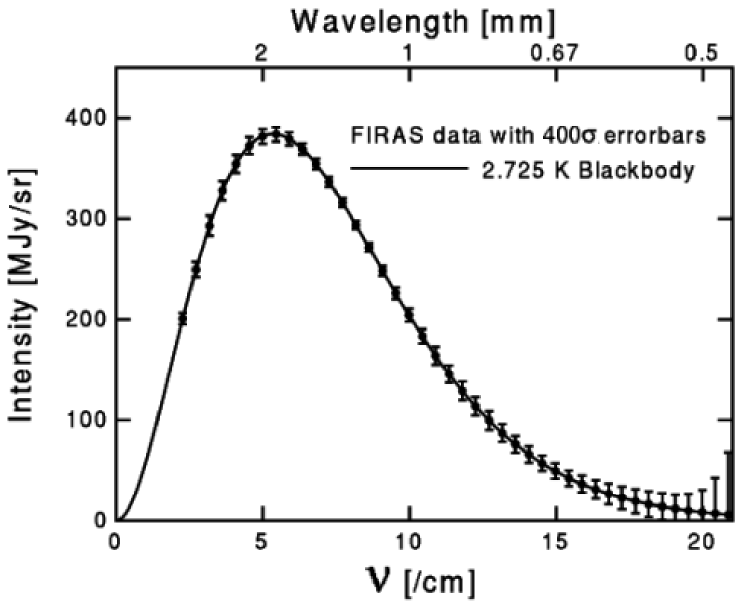
Table 1.1 *Definitions of radiometric quantities*

Symbol	Name	Definition	Units	Equation	Alternate name	Alternate symbol
L_ν	Spectral radiance (frequency units)	Power leaving unit projected surface area into unit solid angle and unit frequency interval	W m^{-2} Hz^{-1} ster^{-1}	$L_\nu = \frac{\varepsilon[2h\nu^3/(c/n)^2]}{\exp(h\nu/kT)-1}$	Specific intensity (frequency units)	I_ν
L_λ	Spectral radiance (wavelength units)	Power leaving unit projected surface area into unit solid angle and unit wavelength interval	W m^{-3} ster^{-1}	$L_\lambda = \frac{\varepsilon[2h(c/n)^2]}{\lambda^5(\exp(hc/\lambda kT)-1)}$	Specific intensity (wavelength units)	I_λ
L	Radiance	Spectral radiance integrated over frequency or wavelength	W m^{-2} ster^{-1}	$L = \int L_\nu d\nu$	Intensity or specific intensity	I
M	Radiant exitance	Power emitted per unit surface area	W m^{-2}	$M = \int L(\theta)d\Omega$		
Φ	Flux	Total power emitted by source of area A	W	$\Phi = \int M dA$	Luminosity	L
E	Irradiance	Power received at unit surface element; equation applies well removed from the source at distance r	W m^{-2}	$E = \frac{\int M dA}{(4\pi r^2)}$		
E_ν, E_λ	Spectral irradiance	Power received at unit surface element per unit frequency or wavelength interval	W m^{-2} Hz^{-1}, W m^{-3}		Flux density	S_ν, S_λ

2. Black Body Radiation

Most astronomical sources emit close to a black body. described either by a single temperature or by a series of temperatures.

Example: COBE measurement of the cosmic background



Black Body Emission

The specific intensity I_ν of a blackbody is given by Planck's law as:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$$

in units of [W m^{-2} sr^{-1} Hz^{-1}]

In terms of wavelengths this corresponds to:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda kT}\right)-1}$$

in units of [W m-1 sr-1]

Note for the conversion frequency ⇔ wavelength:

$$dv = \frac{c}{\lambda^2}d\lambda \quad \text{or} \quad d\lambda = \frac{c}{\nu^2}d\nu$$

Black Body Approximations

At high frequencies (hν >> kT) we get Wien's law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{kT}\right)$$

At low frequencies (hν << kT) we get Rayleigh-Jeans' law:

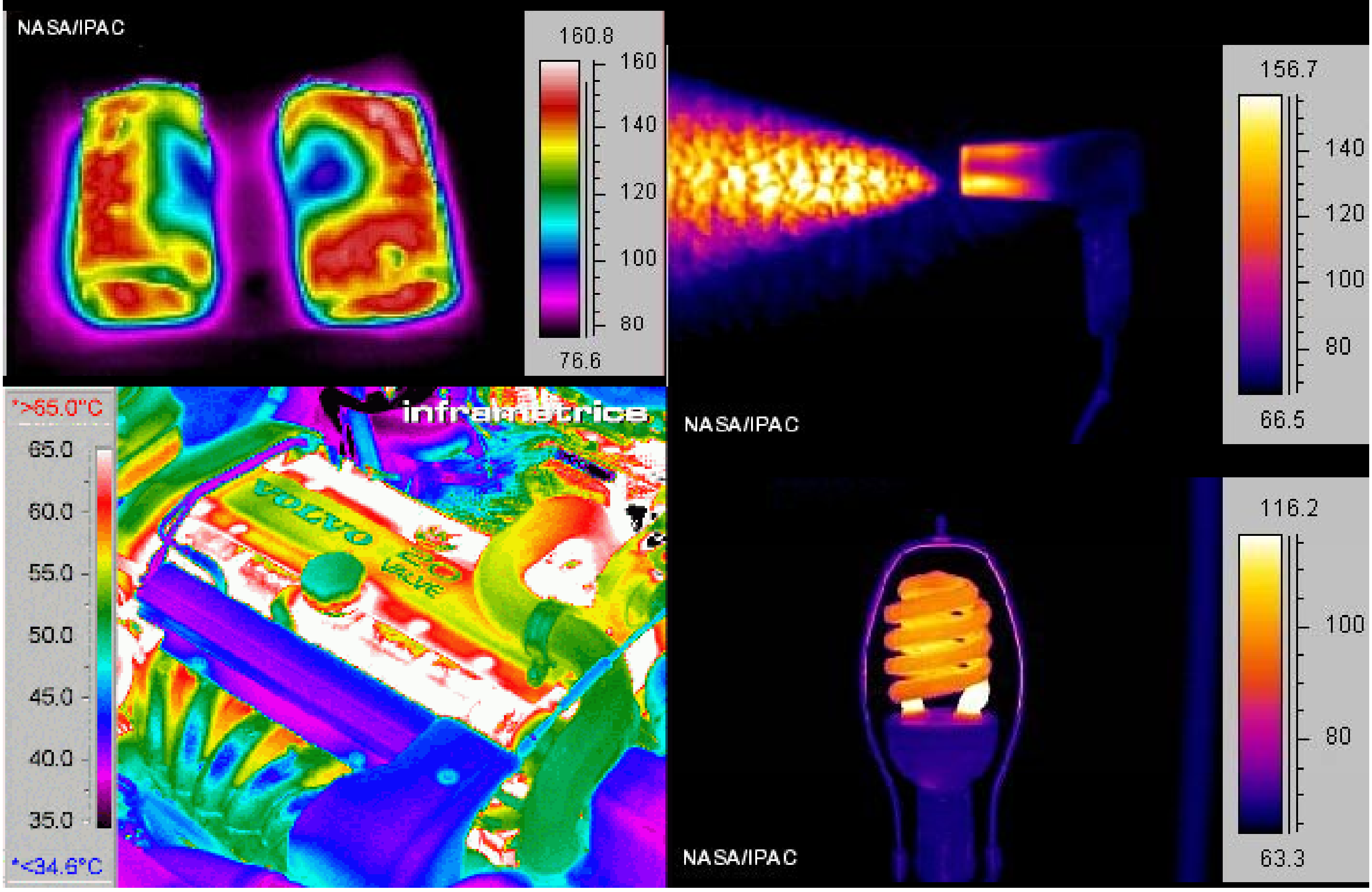
$$B_\nu(T) \approx \frac{2\nu^2}{c^2}kT = \frac{2kT}{\lambda^2}$$

The total radiated power per unit surface is proportional to the fourth power of the temperature:

$$\int_\Omega \int_\nu B_\nu(T)\, d\nu\, d\Omega = \sigma T^4$$

$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzmann constant.

Temperature ⇔ Radiation



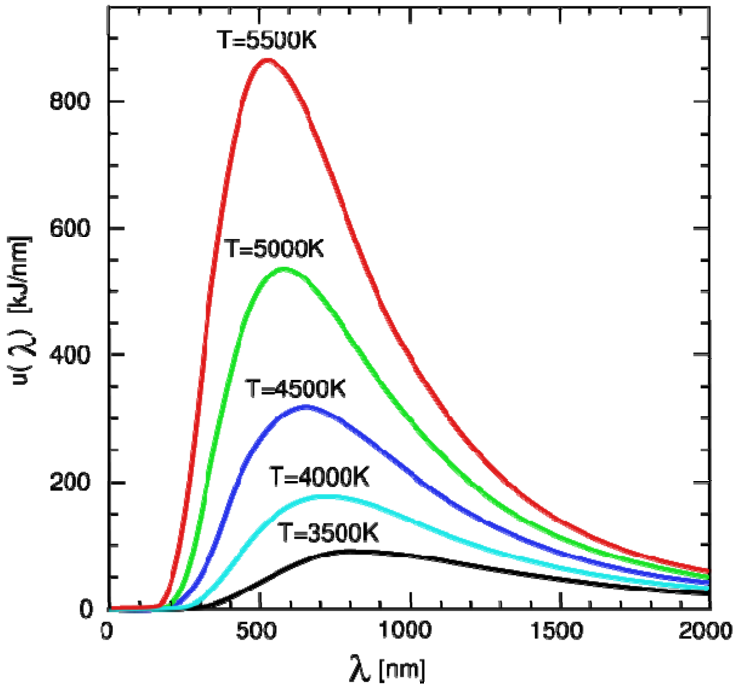
Black Body Temperatures

The temperature corresponding to the maximum specific intensity is given by:

$$\frac{c}{\nu_{max}} T = 5.096 \cdot 10^{-3} \text{ mK} \text{ or } \lambda_{max} T = 2.98 \cdot 10^{-3} \text{ mK}$$

Hence, cooler BBs have their peak emission at longer wavelengths and at lower intensities.

Note: assuming BB radiation one often describes the emission from objects via their effective temperature.



3. Polarization and Coherence

Coherence (from Latin *cohaerere* = to be connected) of electromagnetic waves enables temporally and spatially constant interference.

Ideal case of an uni-directional monochromatic wave (perfect laser): it is possible to define the relative phase at two arbitrary points along **k**.

"Worst" case (in terms of coherence): black-body radiation.

Two types of coherence:

1. spatial coherence → image formation
2. temporal coherence → spectral analysis

Mutual Degree of Coherence

Consider a complex field V(t) as a stationary random process with power spectrum S(ν) and time average <V(t)> = 0.

Measure the fields at any two points in space $V_1(t)$ and $V_2(t)$. The cross correlation between these measurements is given by

$$\Gamma_{12}(\tau) = \langle V_1(t) V_2^*(t+\tau) \rangle$$

Note that the *mean* intensity at point 1 can be described by

$$\Gamma_{11}(0) = \langle V_1(t) V_1^*(t) \rangle$$

The (mutual degree of) coherence can then be defined as:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{[\Gamma_{11}(0)\Gamma_{22}(0)]^{1/2}}$$

Note that γ_{12} includes both spatial (points 1,2) and temporal (τ) coherence.

Quasi-Monochromatic Radiation

Quasi-monochromatic radiation = spectral density is confined to the neighbourhood Δν of some frequency ν:

$$S(\nu) = \exp\left(-\frac{(\nu-\nu_0)^2}{\Delta\nu^2}\right)$$

The autocorrelation of V(t) is then $R(\tau) = \exp\left(-\frac{\tau^2}{\tau_c^2}\right)$

which yields the relation between spectral width Δν and temporal width Δτ:

$$\tau_c \Delta\nu \cong 1$$

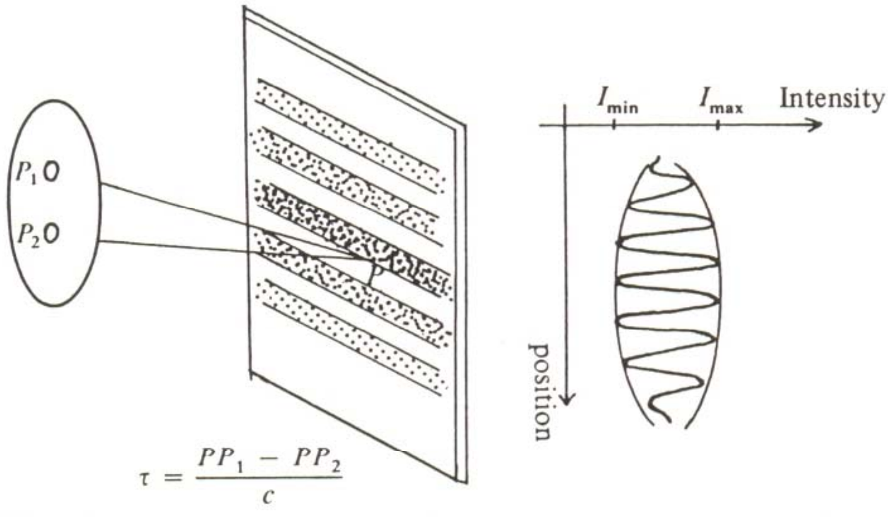
The coherence length l_c is the length over which the field retains the memory of its phase (i.e., the distance beyond which the waves λ and λ+Δλ are out of step by λ):

$$l_c = c\tau_c = \frac{\lambda_0^2}{\Delta\lambda}$$

For $l \ll c\tau_c$ it follows that: $\gamma_{12}(\tau) \sim \gamma_{12}(0) e^{-2i\pi\nu_0\tau}$
and the coherence is determined by $\gamma_{12}(0)$.

Interference Measurements

Consider Young's double slit experiment in which two diffracted, coherent beams interfere.



The maximum and minimum intensities define the visibility V:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

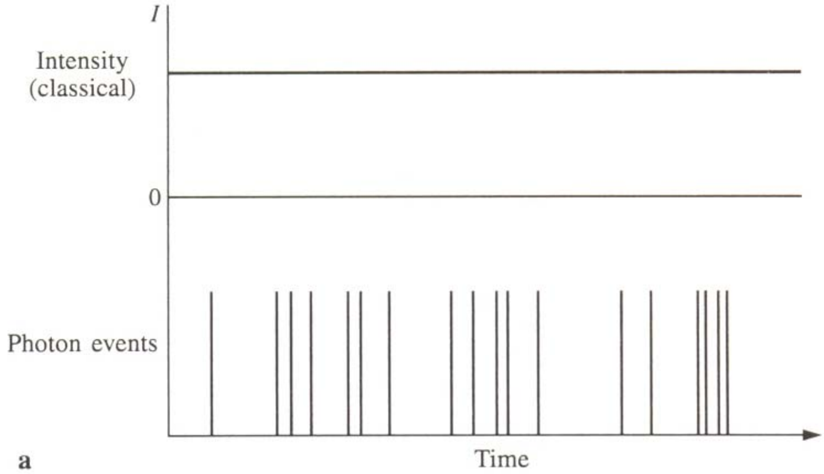
Coherence and Photon Statistics

Now we consider the particle aspect of light. There are several cases:

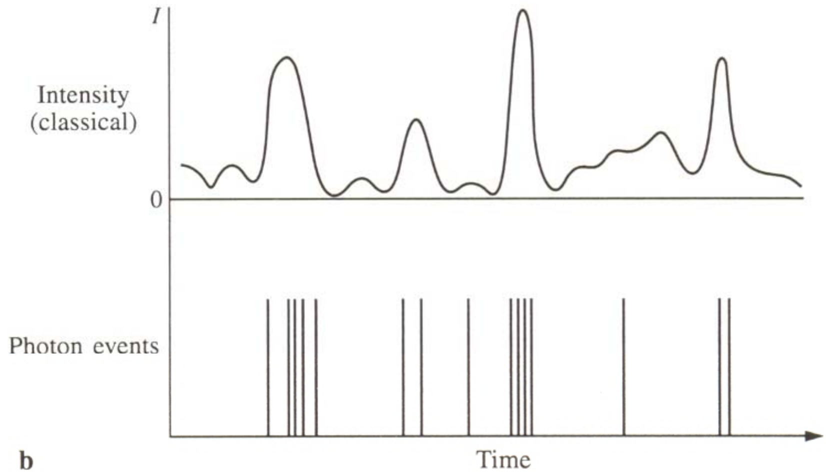
1. Purely monochromatic radiation: τ is infinite. For any τ the number of photons n obeys a Poissonian distribution with variance:
$$\langle \Delta n^2 \rangle = \bar{n}\,\tau$$
2. Quasi-monochromatic radiation: coherence time $\tau_c \sim 1/\Delta\nu$ where $\Delta\nu$ is the line width. If $\tau \gg \tau_c$ the photon fluctuation is given by the Bose-Einstein statistics:
$$\langle \Delta n^2 \rangle = \bar{n}\,\tau\left(1 + \frac{1}{e^{h\nu/kT} - 1}\right)$$
3. For thermal radiation if $\tau \gg \tau_c$ the photons will not follow a Poissonian distribution anymore but group together (bunching) when $\frac{1}{e^{h\nu/kT} - 1} \sim 1$
4. For non-thermal radiation bunching becomes more significant as $\frac{1}{e^{h\nu/kT} - 1}$ increases.

Coherence and Photon Statistics (2)

a) Constant classical intensity and photon events following a Poissonian distribution.



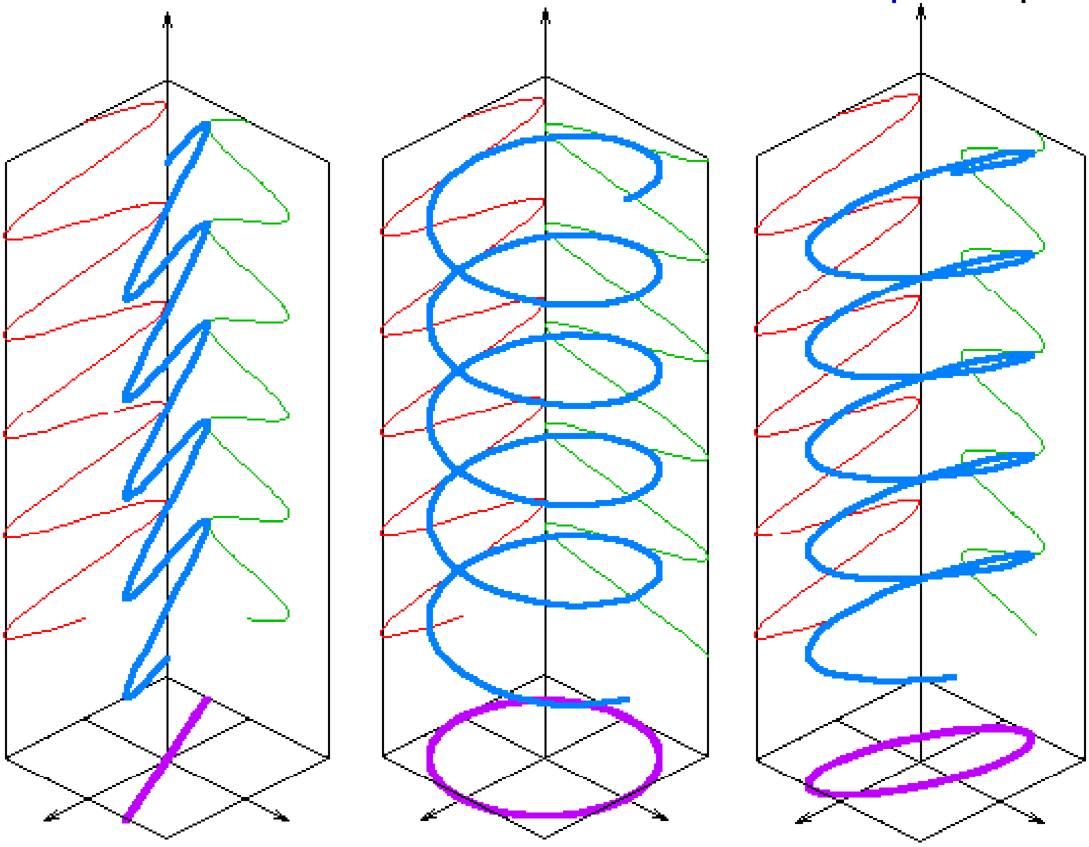
b) Classical intensity of a thermal source with a photon distribution that combines a Poisson process, Bose-Einstein distribution, and bunching.



Polarized Radiation

There are three types of polarized waves:

linear circular and elliptical polarization



Polarized waves

The degree of polarization is important as it carries information on the properties of the source (magnetic fields, dust grain alignment, etc.).

Telescope, instrument optics and detector may alter the polarization.

$$E_x = a_1 \cos(2\pi\nu t - k \cdot r + \phi_1)$$
$$E_y = a_2 \cos(2\pi\nu t - k \cdot r + \phi_2)$$

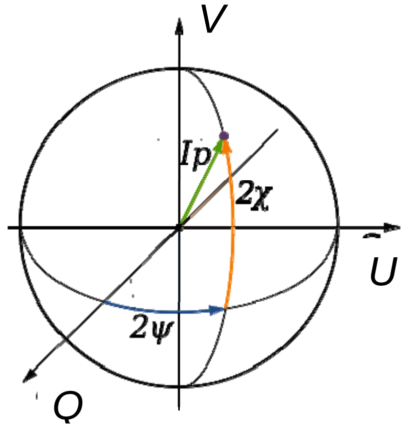
where a_i are the amplitudes, ν is the frequency, k=2π/λ the wavevector, and Φ_i are the phases.

We also define $\Phi = \Phi_2 - \Phi_1$

The Stokes Parameter

Polarization can be defined by the four Stokes parameters I, Q, U, V (1852) as follows:

$$I = a_1^2 + a_2^2$$
$$Q = a_1^2 - a_2^2 = I \cos 2\chi \cos 2\psi$$
$$U = 2a_1 a_2 \cos\phi = I \cos 2\chi \sin 2\psi$$
$$V = 2a_1 a_2 \sin\phi = I \sin 2\chi$$



Generally, the degree of polarization of a wave is:

$$\Pi = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

A plane wave has Π = 1 and the Stokes parameters are related as:

$$I^2 = Q^2 + U^2 + V^2$$

4. Magnitudes

(Apparent) magnitude = *relative* measure of the monochromatic flux e(λ) of a source:

$$m_{\lambda_0} = -2.5\log\frac{e(\lambda_0)}{e_0} = -2.5\log e(\lambda_0) + q_{\lambda_0}$$

The constant q_0 defines magnitude zero.

This system has its origins in the Greek classification of stars according to their visual brightness. The brightest stars were m = 1, the faintest detected with the bare eye were m = 6.

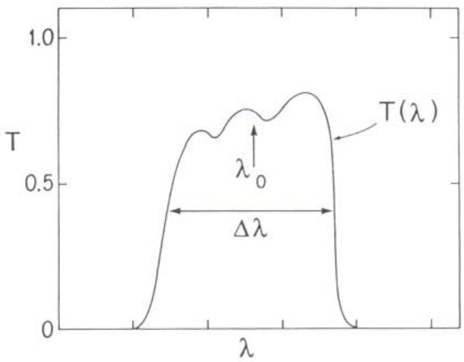
Later formalized by Pogson (1856): a 1st mag star is 100 times brighter than a 6th mag star.

Note: usually 'magnitudes' are being used for quasi-pointlike objects. When referring to surface brightness one uses mag/sr or mag/arcsec^2.

Magnitude systems

In practice, measurements are done through a transmission filter $t_0(\lambda)$ that defines a finite bandwidth:

$$m_{\lambda_0} = -2.5\log\int_0^\infty t_0(\lambda)e(\lambda)d\lambda + 2.5\log\int_0^\infty t_0(\lambda)d\lambda + q_{\lambda_0}$$



As filters differ there are many different photometric systems:

- Johnson UBV system
- Gunn griz
- USNO
- SDSS
- 2MASS JHK
- HST filter system (STMAG)
- ...
- AB magnitude system

m(AB) = -2.5 log(F[W/cm^2/Hz]) - 48.60

Photometric System	Reference
AAO	Allen & Cragg (1983) Elias et al. (1983)
ARNICA	Hunt et al. (1988)
Bessell & Brett	Bessell & Brett (1988)
CIT	Elias et al. (1982) Elias et al. (1983)
ESO	van der Bliek et al. (1996)
Koornneef	Koornneef (1983)
LCO	Persson et al. (1998)
MKO	UKIRT web site (2002)
MSSSO	McGregor (1994)
SAAO	Carter (1990) Carter & Meadows (1995)
UKIRT	Hawarden et al. (2001)

Standard Photometry

Name	λ_0 [µm]	$\Delta\lambda_0$ [µm]	e_0 [W m^{-2} µm^{-1}]	e_0 [Jy]	
U	0.36	0.068	4.35×10^{-8}	1 880	Ultraviolet
B	0.44	0.098	7.20×10^{-8}	4 650	Blue
V	0.55	0.089	3.92×10^{-8}	3 950	Visible
R	0.70	0.22	1.76×10^{-8}	2 870	Red
I	0.90	0.24	8.3×10^{-9}	2 240	Infrared
J	1.25	0.30	3.4×10^{-9}	1 770	Infrared
H	1.65	0.35	7×10^{-10}	636	Infrared
K	2.20	0.40	3.9×10^{-10}	629	Infrared
L	3.40	0.55	8.1×10^{-11}	312	Infrared
M	5.0	0.3	2.2×10^{-11}	183	Infrared
N	10.2	5	1.23×10^{-12}	43	Infrared
Q	21.0	8	6.8×10^{-14}	10	Infrared

1 Jy = 10^{-26} W m^{-2} Hz^{-1}.

Bolometric Magnitude

Bolometric magnitude = integral of the monochromatic flux over all wavelengths:

$$m_{bol} = -2.5\log\frac{\int_0^\infty e(\lambda)d\lambda}{e_{bol}}$$

with e_{bol} = 2.52·10^{-8} W/m^2

If the source radiates isotropically one gets:

$$m_{bol} = -0.25 + 5\log D - 2.5\log\frac{L}{L_\Theta}$$

where L_o = 3.827·10^{26} W is the luminosity of the Sun.

Absolute Magnitude and Color Indices

Absolute magnitude = apparent magnitude of the source if it were at a distance of D = 10 parsecs.

Including a term A for interstellar absorption we get:

$$M = m + 5 - 5\log D - A$$

Color indices = difference of magnitudes at different wavebands = ratio of fluxes at different wavelengths.

- The color indices of an A0 dwarf star are about zero longward of V.
- The color indices of a blackbody in the Rayleigh-Jeans tail are:

  B-V=-0.46, U-B=-1.33, V-R = V-I = ... = V-N = 0.0

Applications: e.g., Color-Color-Diagram