Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

4th Lecture: 24 September 2012

$$S_{cont} = \frac{\sigma h \lambda \sqrt{n_{pix}} 10^{30}}{SR\Delta\lambda A_{tel}\eta_D G \eta_{atm}\eta_{tot} t_{int}} \sqrt{\frac{2hc^2}{\lambda^5}\left(\frac{\varepsilon_T}{\exp\left[\frac{hc}{kT_T\lambda}\right]-1} + \frac{\varepsilon_A}{\exp\left[\frac{hc}{kT_A\lambda}\right]-1}\right)\eta_{tot} \cdot}$$
$$\cdot \sqrt{2\pi\left(1-\cos\left(\arctan\left(\frac{1}{2F\#}\right)\right)\right)D^2{}_{pix} \cdot \frac{\eta_D G\lambda}{hc} \cdot \Delta\lambda \cdot t_{int} + I_d t_{int} + N^2_{read} n}$$

1. Noise: Introduction
2. Noise: Distributions
3. Signal-to-noise (= f{t_{int}, D_{tel}})
4. Instrument sensitivities

Introduction

What is noise?

Wikipedia:

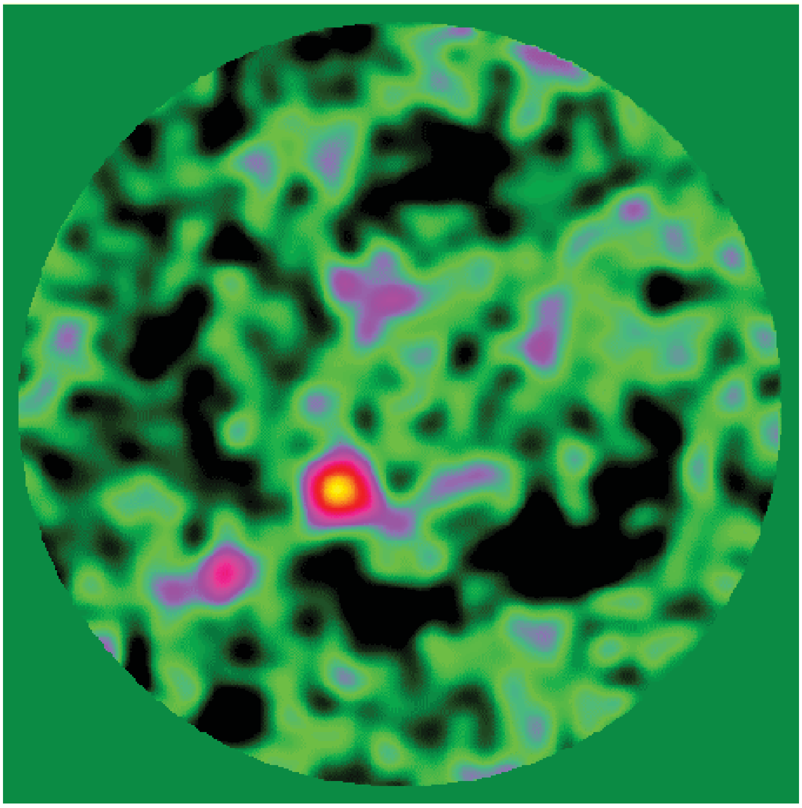
In common use, the word noise means any unwanted sound.

In signal processing or computing it can be considered random unwanted data without meaning.

"Signal-to-noise ratio" is sometimes used to refer to the ratio of useful to irrelevant information in an exchange.

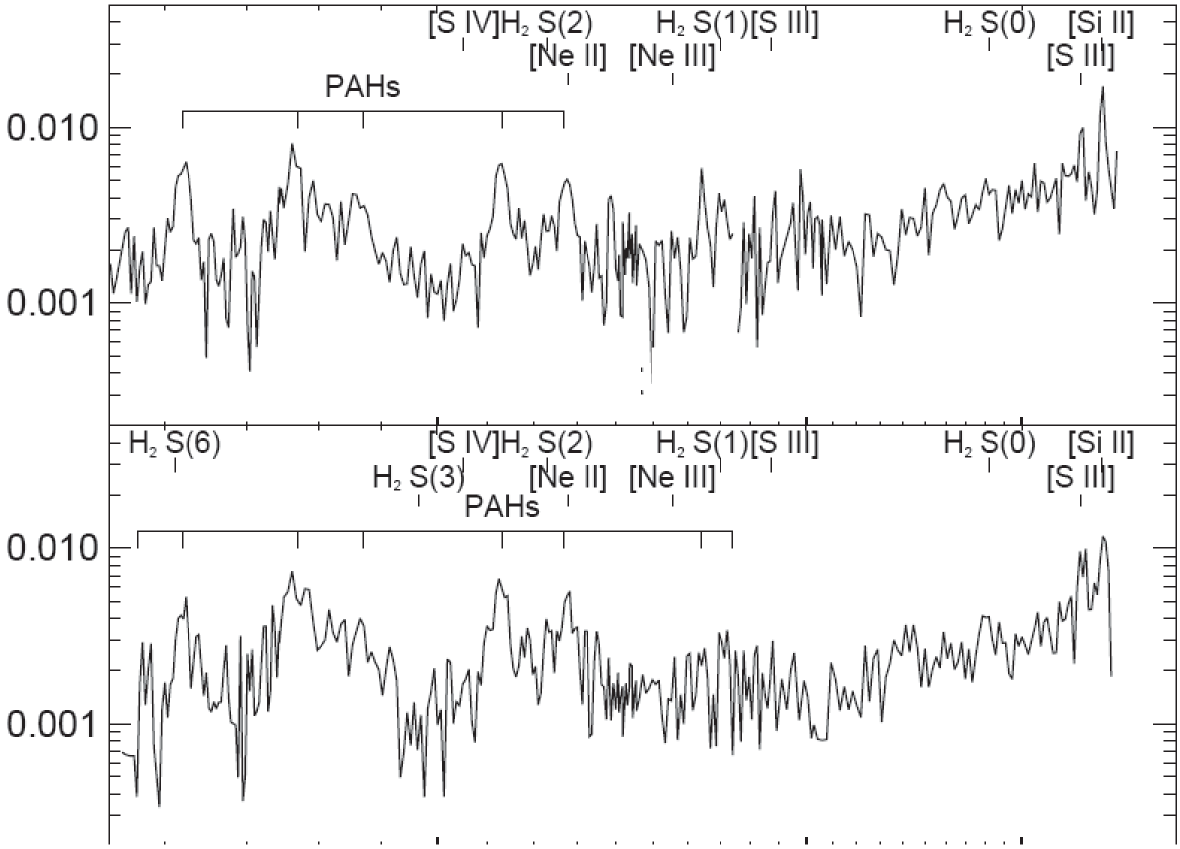
NASA researchers at Glenn Research Center conducting tests on aircraft engine noise in 1967

What is Noise? And what is real Signal?



SCUBA 850μm map of the Hubble deep field

What is Noise? And what is real Signal?



Example: Digitization Noise

Digitization = converting an analog signal into a digital signal using an Analog-to-Digital Converter (ADC).

The number of bits determines the dynamic range of the ADC. The resolution is 2^n, where n is the number of bits.

Typical ADCs have:

12 bit: 2^{12} = 4096 quantization levels

16 bit: 2^{16} = 65636 quantization levels

Too few bits → discrete, "artificial" steps in signal levels → noise

Some Sources of Noise in Astronomical Data

Noise type	Signal	Background
Photon shot noise	X	X
Scintillation	X	
Cosmic rays		X
Image stability	X	
Read noise	X	X
Dark current noise	X	X
CTE (CCDs)	X	X
Flat fielding (non-linearity)	X	X
Digitization noise	X	X
Other calibration errors	X	X
Image subtraction	X	X

Noise Distribution

Noise Distribution: 1.Gaussian Noise

Gaussian noise is the noise following a Gaussian (normal) distribution.

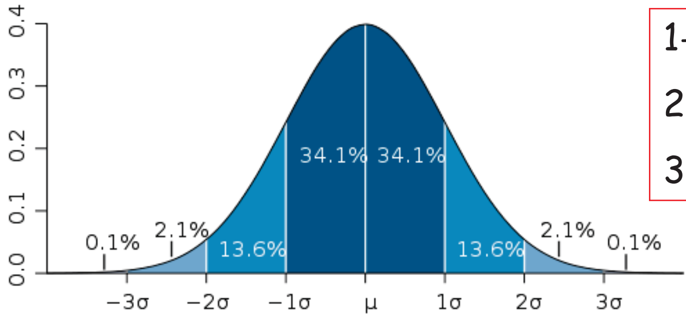
It is often (incorrectly) called white noise, which refers to the (un-)correlation of the noise.

$$S = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

x is the actual value

μ is the mean of the distribution

σ is the standard deviation of the distribution



1-σ ~ 68%

2-σ ~ 95%

3-σ ~ 99.7%

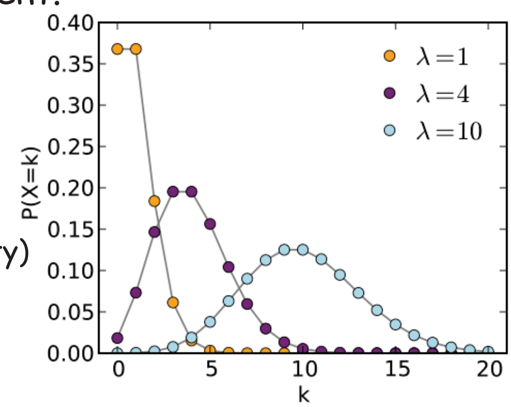
Astronomers usually consider S/N > 3σ as significant.

Noise Distribution: 2. Poisson Noise

Poisson noise is the noise following a Poissonian distribution.

It expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known *average rate* and *independently* of the time since the last event.

$$P(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$



k is the number of occurrences of an event (probability)

λ is the *expected* number of occurrences

- the mean (average) of $P(k,\lambda)$ is λ.
- the standard deviation of $P(k,\lambda)$ is $\sqrt{\lambda}$.

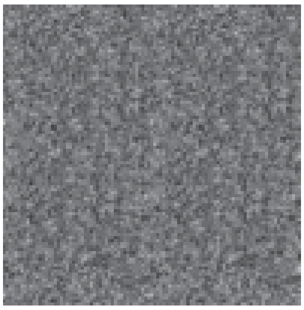
Example: fluctuations in the detected photon flux between finite time intervals Δt. Detected are k photons, while expected are on average λ photons.

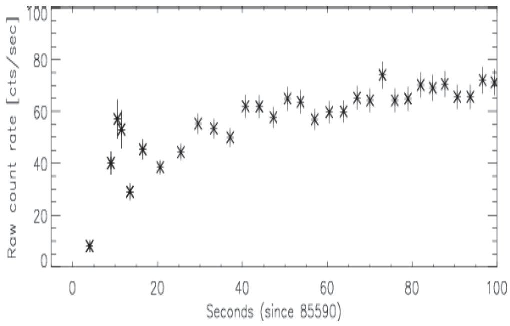
Side note on Noise Measurement

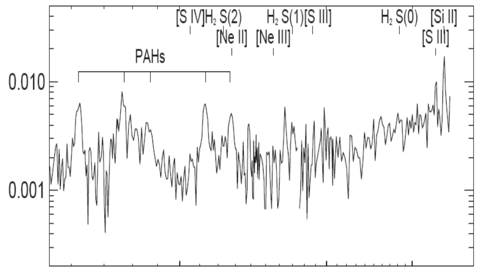
Let's assume the noise distribution is purely Gaussian or Poissonian and no other systematic noise is present.

Then the spatial distribution (neighbouring pixels) of the noise is equivalent to the temporal distribution (successive measurements with one pixel)

This is analogous to throwing 5 dices once versus throwing one dice 5 times.







Case 1: Spatial noise (detector pixels)

Case 2: Repeated measurements in time (time series)

Case 3: Spectrum (dispersed information)

Poisson Noise and Integration Time

We integrate light from a uniformly extended source on our camera CCD.

In the finite time interval Δt we expect an average of Λ photons. Due to the statistical nature of the photon arrival rate some pixels will detect more, some less than Λ photons.

The noise on the signal Λ (i.e., between pixels) is $\sqrt{\Lambda}$

Now we integrate for $2\times\Delta t$ → we expect an average of $2\times\Lambda$ photons.

The noise on that signal is now $\sqrt{(2\times\Lambda)}$, i.e., it has increased by $\sqrt{2}$

Generally speaking, with respect to the integration time, the noise will only increase $\sim\sqrt{t}$ while the signal increases $\sim t$.

The Signal-to-Noise (S/N)

S/N Basics

Wikipedia:

Signal-to-noise ratio (often abbreviated SNR or S/N) is a measure used in science and engineering that compares the level of a desired signal to the level of background noise.

Signal = S; Background = B; Noise = N;

$$\sigma = \frac{\text{Signal}}{\text{Noise}}$$

← measured as (S+B) - mean{B}

← total noise = $\sqrt{\sum (N_i)^2}$ (if statistically independent)

Both S and N should be in units of events (photons, electrons, data numbers) per unit area (pixel, PSF size, arcsec^2).

S/N and Integration Time

Assuming the signal suffers from Poisson shot noise. Let's calculate the dependence on integration time t_{int}:

Integrating t_{int}: $$\sigma = \frac{S}{N}$$

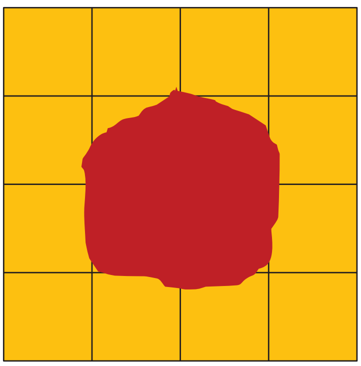
Integrating $n \times t_{int}$: $$\sigma = \frac{n \cdot S}{\sqrt{n \cdot B}} \overset{N=\sqrt{B}}{=} \sqrt{n}\,\frac{S}{N} \quad \Rightarrow \quad \frac{S}{N} \propto \sqrt{t_{\mathrm{int}}}$$

You need to integrate four times as long to get twice the S/N.

Several Cases to Consider...

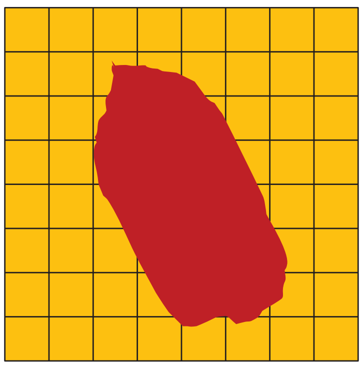
Background (=noise)

Target



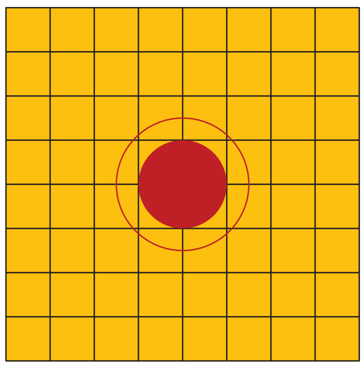
Seeing-limited case

- pixel size ~ seeing
- PSF ≠ f{D}



Diffraction-limited, extended source

- pixel size ~ diff.lim
- PSF = f{D}
- target >> PSF



Diffraction-limited, point source

- pixel size ~ diff.lim
- PSF = f{D}

Case 1: Seeing-limited "Point Source"

Signal = S; Background = B; Noise = N; Telescope diameter = D

θ_{seeing} ~ const

If detector is Nyquist-sampled to θ_{seeing} :

$S \sim D^2$ (area)

$B \sim D^2$ → $N \sim D$ (Poisson std.dev)

➔ $S/N \sim D$

➔ $t_{int} \sim D^{-2}$

Case 2: Diffraction-limited extended Source

Signal = S; Background = B; Noise = N; Telescope diameter = D

"PSFØ" ~ const

If detector Nyquist sampled to θ_{diff}: pixel ~ D^{-2} but $S \sim D^2$

D^2 (telescope size) and D^{-2} (pixel FOV) cancel each other
→ no change in signal

same for the background flux

➔ S/N ~ const → t_{int} ~ const → *no gain for larger telescopes!*

Case 2B: offline re-sampling by a factor x (makes θ_{diff} x-times larger)

since $S/N \sim \sqrt{n_{pix}}$ → $S/N \sim \sqrt{x^2} = x$ → $t_{int} \sim x^{-2}$.

Case 3: Diffraction-limited "Point Source"

Signal = S; Background = B; Noise = N; Telescope diameter = D

"$S/N = (S/N)_{light\ bucket} \cdot (S/N)_{pixel\ scale}$"

(i) Effect of telescope aperture:

$S \sim D^2$

$B \sim D^2 \rightarrow N \sim D$ $\rightarrow S/N \sim D$

(ii) Effect of pixel FOV (if Nyquist sampled to θ_{diff}):

$S \sim$ const (pixel samples PSF = all source flux)

$B \sim D^{-2} \rightarrow N \sim D^{-1}$ $\rightarrow S/N \sim D$

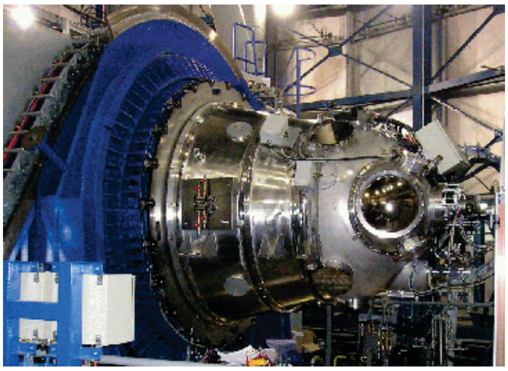
(i) and (ii) combined $S/N \sim D^2 \rightarrow t_{int} \sim D^{-4}$

→ *huge gain: 1hr ELT = 3 months VLT*

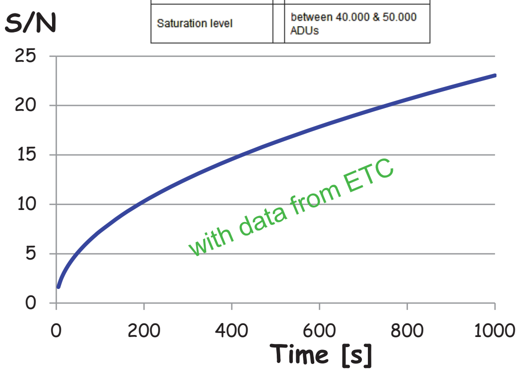
Instrument Sensitivities

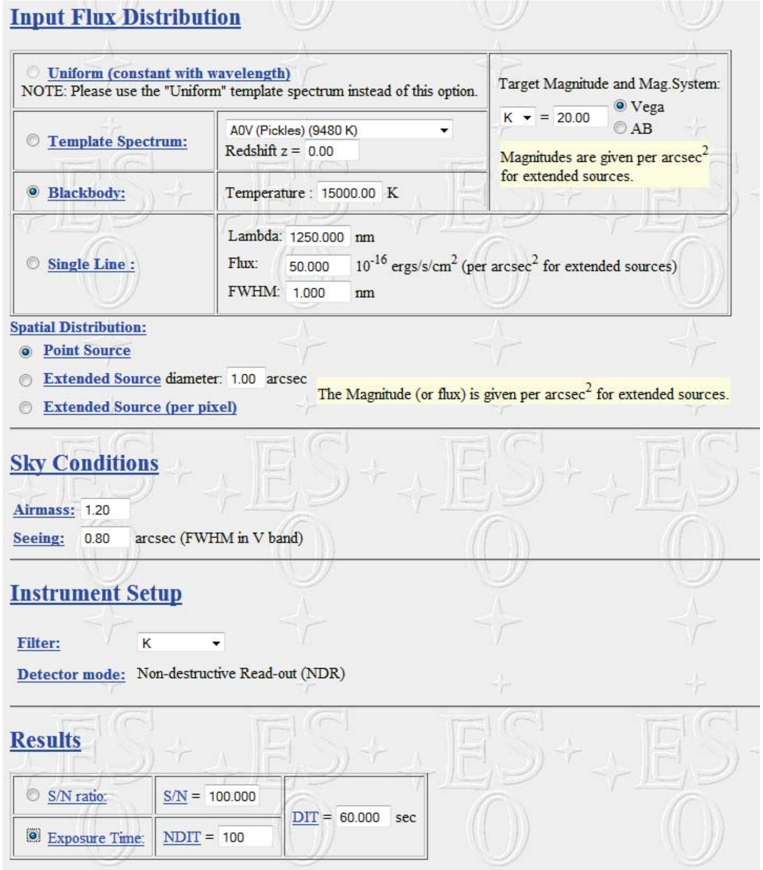
Example: ESO's HAWK-I Wide Field Imager

http://www.eso.org/observing/etc/bin/ut4/hawki/script/hawkisimu



Operating temperature	75K, controlled to 1mK
Dark current [e-/s] (at 75K)	between 0.10 & 0.15
Read noise* (DCR)	~ 12 e-
Read noise* (NDR)	~ 5 e-
Linear range (1%)	60.000e- (~30.000 ADUs)
Saturation level	between 40.000 & 50.000 ADUs





Preface

(i) in this discussion we neglect quantum (shot) noise from the source.

(ii) we consider only point sources.

$$\sigma = \frac{S_{el}}{N_{tot}}$$

detected signal

total noise

number of pixels

background noise

background flux

read noise

dark current

intensity

pixel FOV

...

Note: There is no "one fits all" recipe!

Detected Signal

The detected signal S_{el} depends on:

- the source flux density S_{src} [photons s^{-1} cm^{-2} μm^{-1}]
- the integration time t_{int} [s]
- the telescope aperture A_{tel} [m^2]
- the transmission of the atmosphere η_{atm}
- the total throughput of the system η_{tot}, which includes:
  - the reflectivity of all telescope mirrors
  - the reflectivity (or transmission) of all instrument components, such as mirrors, lenses, filters, beam splitters, grating efficiencies, slit losses, etc.
- the Strehl ratio SR
- the detector responsivity $\eta_D G$
- the spectral bandwidth $\Delta\lambda$ [µm]

$$S_{el} = S_{src} \cdot SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{\mathrm{int}}$$

Total Noise (1)

The total noise N_{tot} depends on:

- the number of pixels n_{pix} of one resolution element
- the background noise per pixel N_{back}

$$N_{tot} = N_{back} \sqrt{n_{pix}}$$

where the total background noise N_{back} depends on:

- the background flux density S_{back}
- the integration time t_{int}
- the detector dark current I_d
- the pixel read noise (N) and detector frames (n)

$$N_{back} = \sqrt{S_{back} \cdot t_{\mathrm{int}} + I_d \cdot t_{\mathrm{int}} + N_{read}^2 \cdot n}$$

Total Noise (2)

The background flux density S_{back} depends on:

- the total background intensity $B_{tot} = (B_T + B_A) \cdot \eta_{tot}$
  where B_T and B_A are the thermal emissions from telescope and atmosphere, approximated by black body emission $B_{T,A} = \frac{2hc^2}{\lambda^5} \left[\frac{\varepsilon}{\exp\left[\frac{hc}{kT\lambda}\right] - 1} \right]$
- the spectral bandwidth $\Delta\lambda$
- the pixel field of view $A \times \Omega = 2\pi \left(1 - \cos\left(\arctan\left(\frac{1}{2F\#} \right) \right) \right) D^2_{pix}$
- the detector responsivity $\eta_D G$, and
- the photon energy hc/λ

$$S_{back} = B_{tot} \cdot A \times \Omega \cdot \frac{\eta_D G \lambda}{hc} \cdot \Delta\lambda$$

Resulting Instrument Sensitivity

Putting it all together, the minimum detectable source signal is:

$$\sigma = \frac{S_{el}}{N_{tot}} = \frac{S_{src} \cdot SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{\text{int}}}{N_{back} \sqrt{n_{pix}}}$$

$$\Rightarrow S_{src} = \frac{\sigma \cdot \sqrt{S_{back} \cdot t_{\text{int}} + I_d \cdot t_{\text{int}} + N_{read}^2 \cdot n} \cdot \sqrt{n_{pix}}}{SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{\text{int}}}$$

Now we can calculate the unresolved line sensitivity S_{line} [W/m²] from the source flux S_{src} [photons/s/cm²/µm]:

$$S_{line} = \frac{hc}{\lambda} S_{src} \Delta\lambda \cdot 10^4$$

and with the relation $S_\lambda \left[\frac{W}{m^2 \mu m} \right] = S_\nu [Jy] \cdot 10^{-26} \frac{c}{\lambda^2}$

we can calculate the continuum sensitivity S_{cont}:

$$S_{cont} = \frac{hc}{\lambda} S_{src} \cdot 10^4 \cdot \frac{\lambda^2}{c} \cdot 10^{26} = 10^{30} h \lambda S_{src}$$