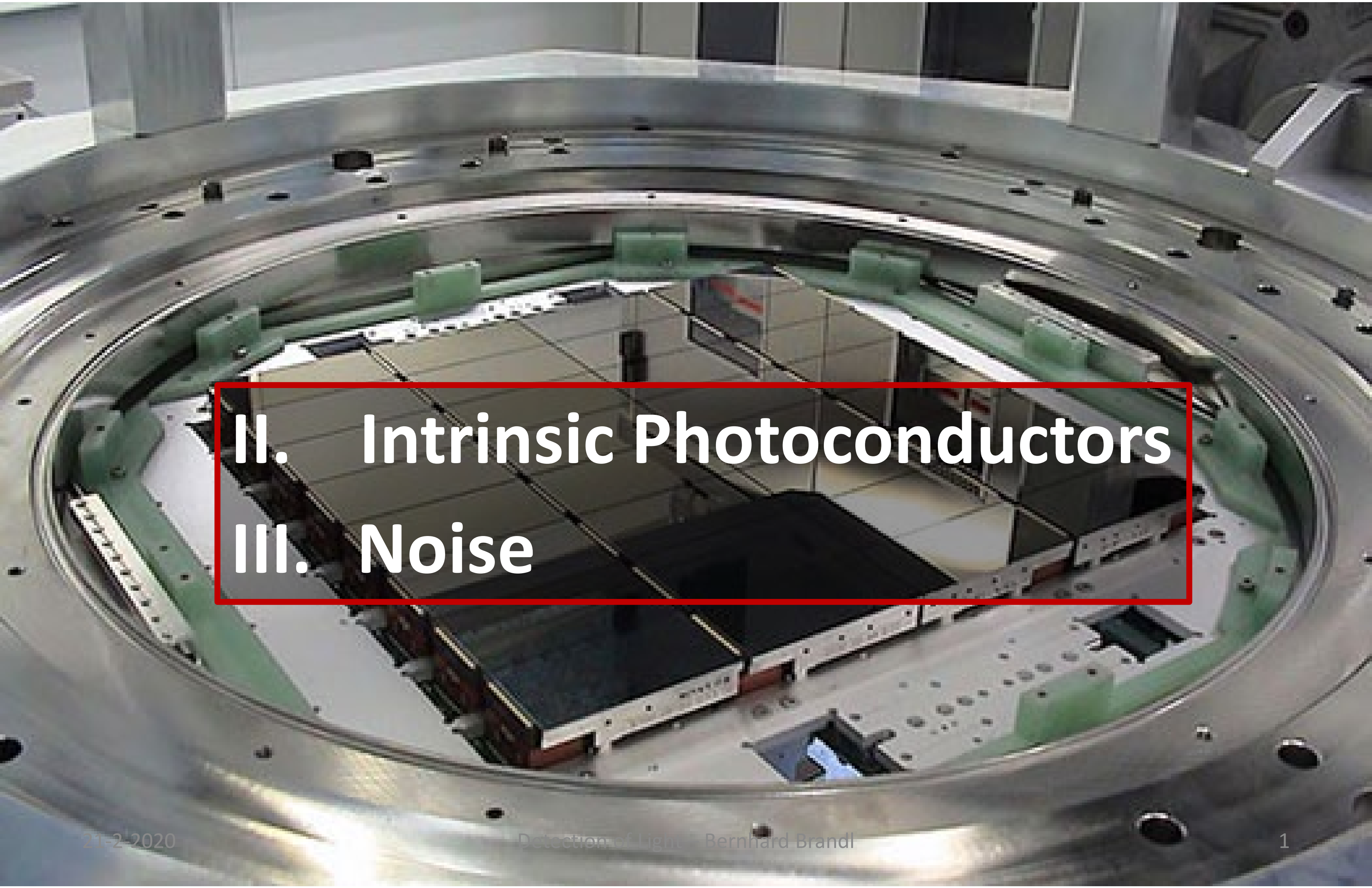
# Detection of Light



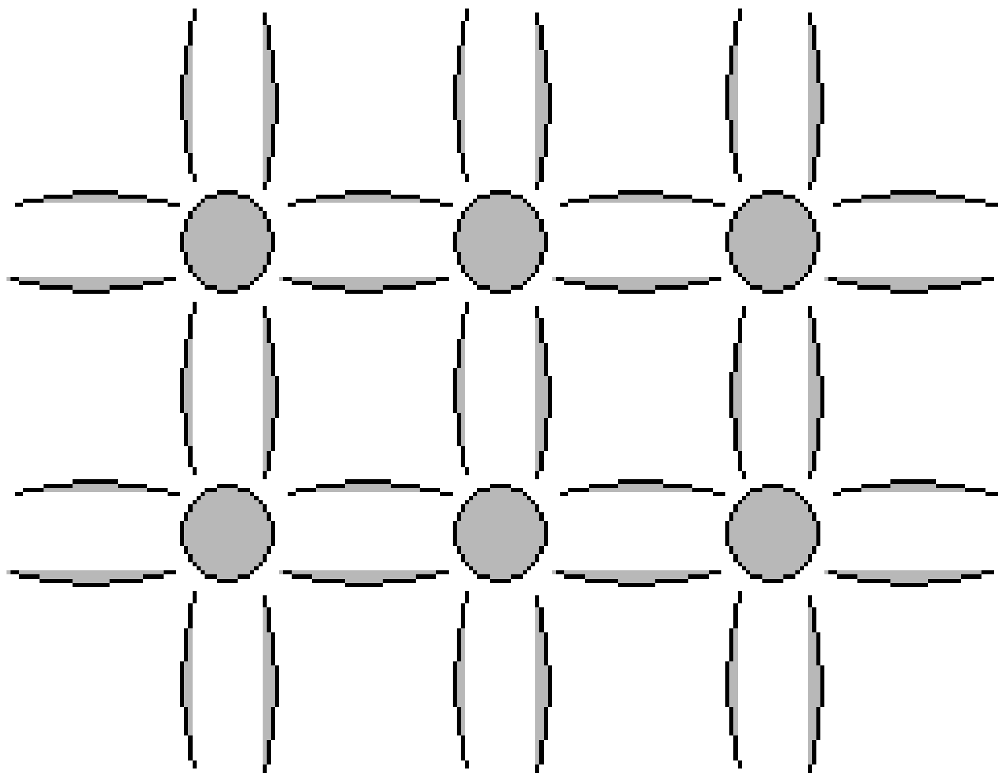
II. Intrinsic Photoconductors

III. Noise

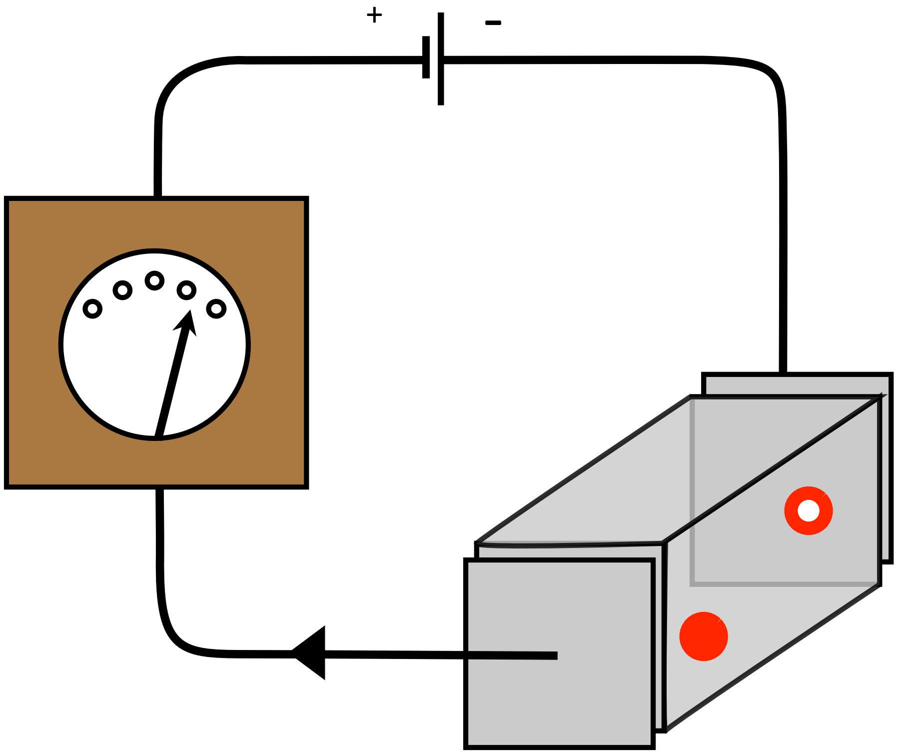
# Fundamental Principle

# Basic Principle – Physics

$E_\gamma$ lifts $e^-$ from valence into conduction band: $E_\gamma = \frac{hc}{\lambda} > E_{bandgap}$



# Basic Principle – Realization



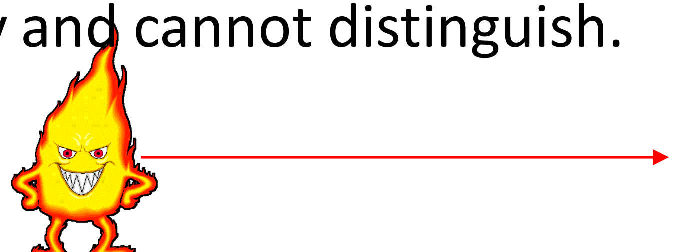

Applying an electric field causes electric charges to move in the material and register a signal as an electric current.

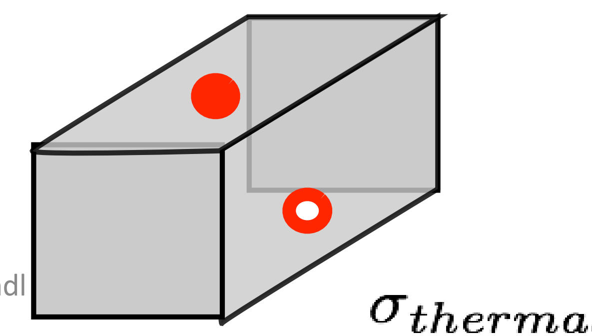
# Practical Limitations

- Wavelength cutoffs: $$\lambda_c = \frac{hc}{E_g}$$

→ Germanium (0.67 eV): 1.85µm
→ Silicon (1.14 eV): 1.12µm
→ GaAs (1.43 eV): 0.87µm

- Cleanliness and non-uniformity of material
- Problems to make good electrical contacts to pure Si
- Charge carriers are generated by both photons and thermal excitation ("dark current"). We only measure the electrical conductivity and cannot distinguish.

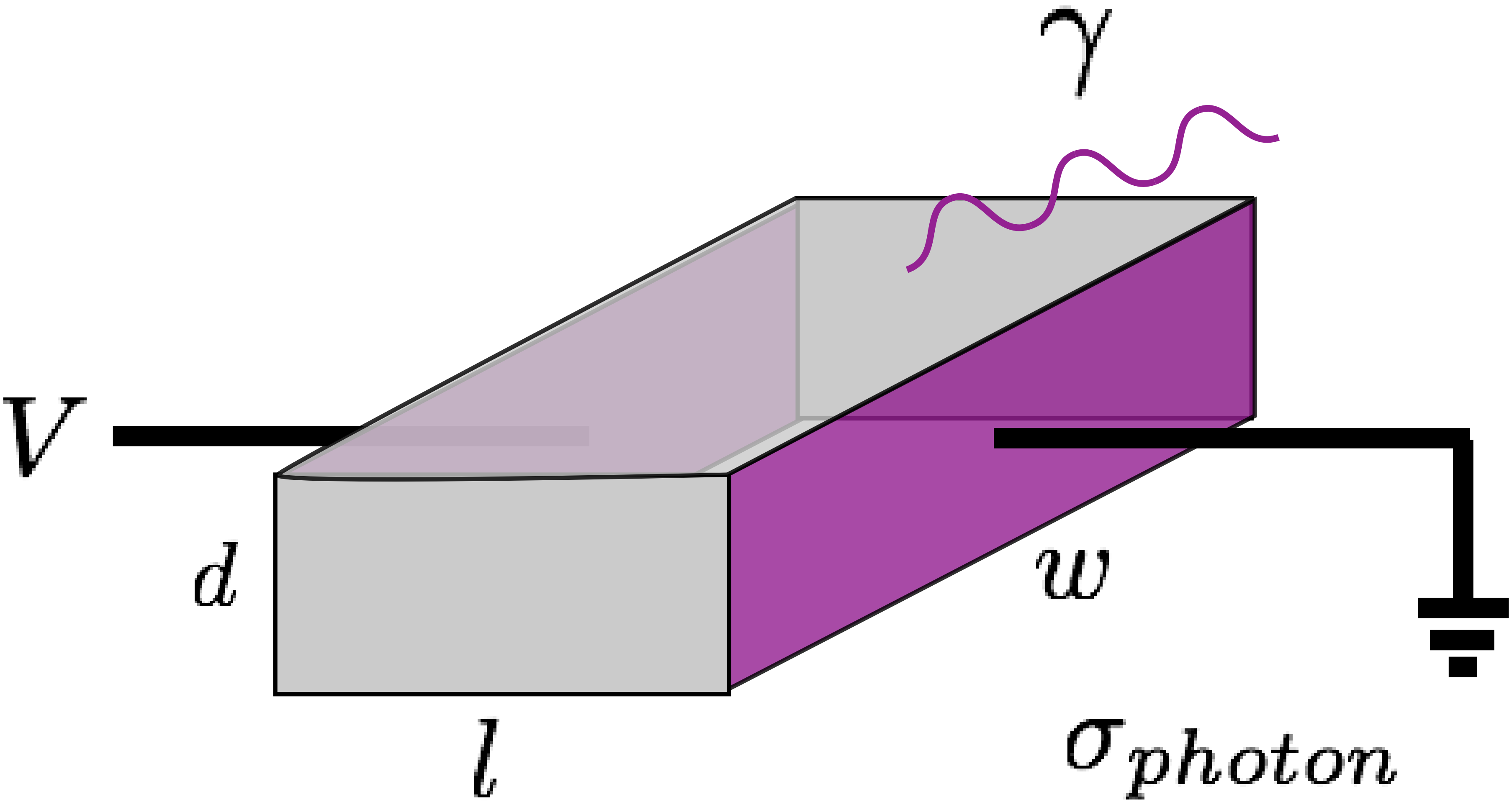




# Basic Electric Properties

# Schematics of a Detector

Consider a pixel with physical dimensions *d, l, w*:



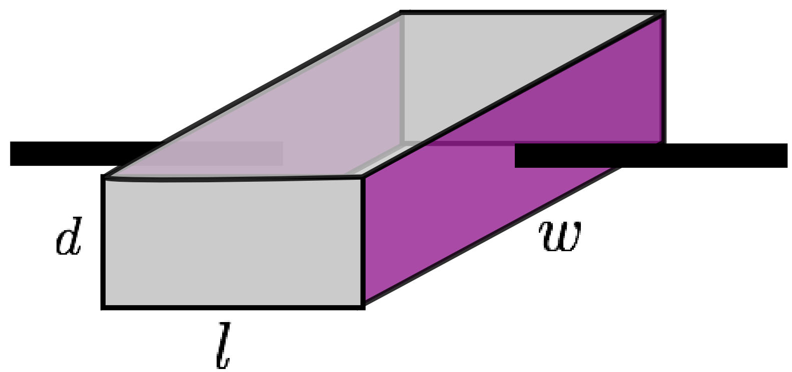
# Resistivity, Resistance, and Conductivity

Resistance $R$ [ Ω] ≡ measure of a material's opposition to the flow of electric current

$$R \equiv \frac{U}{I}$$

Resistivity $\rho$ [ Ω m ] ≡ intrinsic material property to oppose the flow of an electric current.

$$R = \frac{\rho \cdot l}{A} = \frac{\rho \cdot l}{d \cdot w}$$




→ Resistivity:

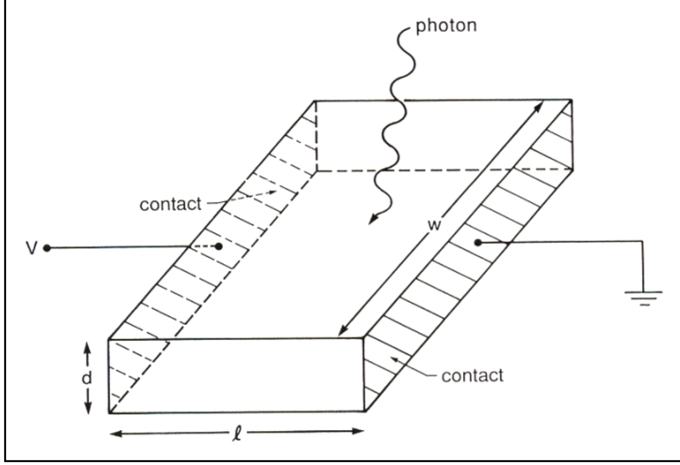
$$\rho = R \cdot \frac{d \cdot w}{l}$$

Conductivity σ is the inverse of the resistivity:

$$\sigma = \frac{1}{\rho}$$

# Deriving the “Photo-Current” (1)

# Conductivity σ ⇔ Photon Flux




Ohm's law: $R_d = \frac{V_b}{I_d}$

The conductivity σ [$\Omega^{-1}\,cm^{-1}$] is related to $R_d$ via: $\sigma = \frac{1}{R_d}\frac{l}{A} \Rightarrow R_d = \frac{1}{\sigma}\frac{l}{wd}$

where: $\sigma = \sigma_{th} + \sigma_{ph} \approx \sigma_{ph}$ (`*th*' denotes the thermal "dark" current)

The current density is: $J_x = q_c n_0 \langle v_x \rangle$

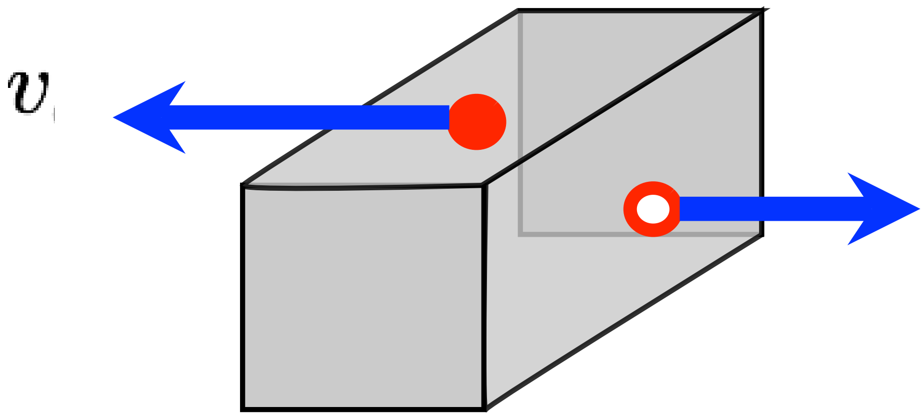
where $q_c$ is the electrical charge and $n_o$ the carrier density

But also: $J_x = \frac{I}{A} = \frac{I_d}{wd} = \frac{V_b}{R_d wd} = \frac{\sigma V_b}{l} = \sigma E_x$

and we get: $q_c n_0 \langle v_x \rangle = \sigma E_x$

# Electron Mobility

# Electron Mobility




Mean drift velocity $\langle v_x \rangle$ [m/s] depends on the electric field $E$ [V/m] and the electron mobility $\mu$ [cm$^2$/(Vs)]:

$$\langle v_x \rangle = \mu E_x$$

With that, and $q_c n_0 \langle v_x \rangle = \sigma E_x$, we can calculate the conductivity σ:

$$\sigma \overset{q_c=-q}{=} \frac{-q n_0 \langle v_x \rangle}{E_x} \overset{-\mu_n = \frac{\langle v_x \rangle}{E_x}}{=} q n_0 \mu_n$$

# Electron Mobility = f{T}

*Mobility ~ mean time between collisions*

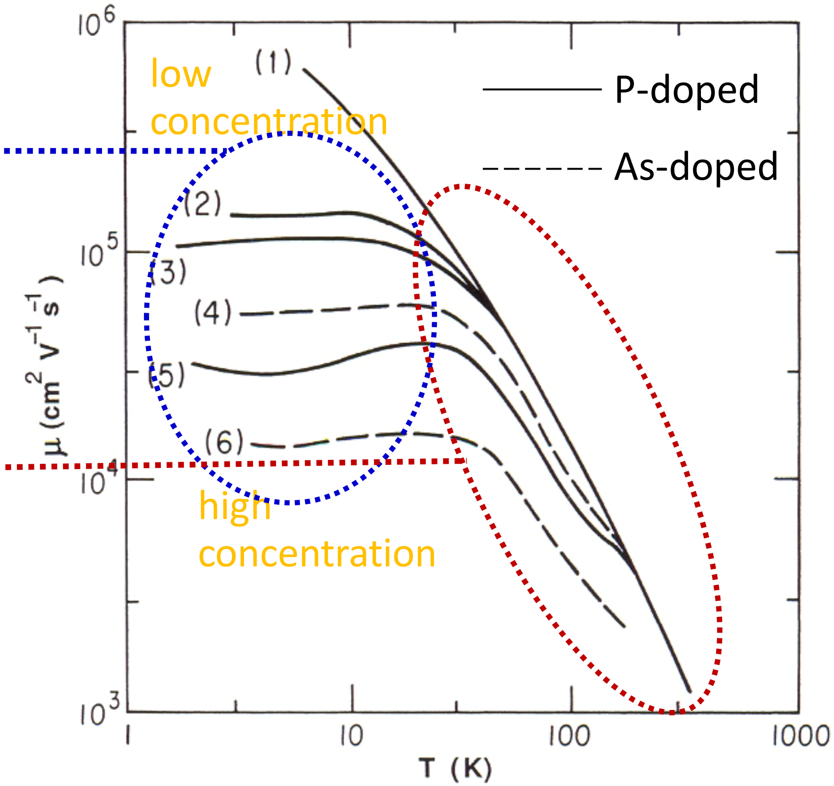
@low *T* → impurities dominate

ionized impurities: $\mu_n \sim T^{3/2}$

neutral impurities: $\mu_n \sim$ const.

@high *T* → crystal lattice dominates

$\mu_n \sim T^{-3/2}$




*Astronomical detectors usually operate in the low T regime.*

# Typical Electron Mobility Numbers

Table 3 Carrier mobilities at room temperature, in $cm^2/V\text{-}s$

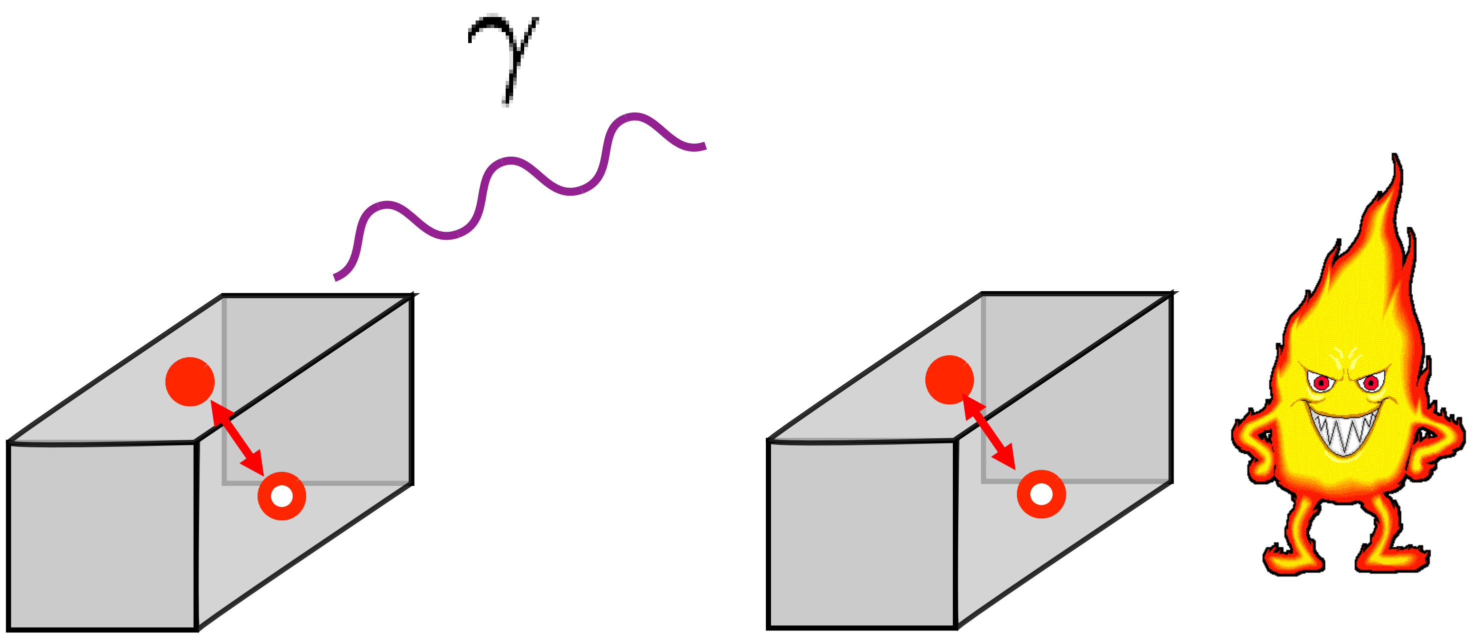
| Crystal | Electrons | Holes | Crystal | Electrons | Holes |
|---|---|---|---|---|---|
| Diamond | 1800 | 1200 | GaAs | 8000 | 300 |
| Si | 1350 | 480 | GaSb | 5000 | 1000 |
| Ge | 3600 | 1800 | PbS | 550 | 600 |
| InSb | 800 | 450 | PbSe | 1020 | 930 |
| InAs | 30000 | 450 | PbTe | 2500 | 1000 |
| InP | 4500 | 100 | AgCl | 50 | — |
| AlAs | 280 | — | KBr (100 K) | 100 | — |
| AlSb | 900 | 400 | SiC | 100 | 10–20 |

*Generally, holes are less mobile than electrons,*
*but there are exceptions.*

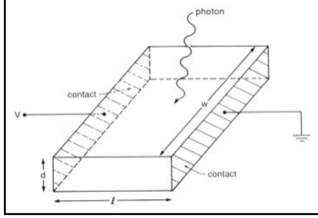
# Deriving the “Photo-Current” (2)

# Mean Lifetime for the Charge Carriers

Eventually, the electrons and holes recombine after a mean lifetime $\tau$, releasing the energy either as heat or light.



# Conductivity σ ⇔ Photon Flux




To the total conductivity, both electrons and holes contribute:

$$\sigma_{ph} = q\left(\mu_n n + \mu_p p\right)$$

(*n* and *p* are the negative and positive charge carrier concentrations)

Now, consider the incoming photon flux ϕ [γ/s]

→ The number of charge carriers in equilibrium is $\varphi\eta\tau$, where $\eta$ is the quantum efficiency, and $\tau$ is the mean lifetime before recombination. Typically, $\tau \sim$ (impurity concentration)$^{-1}$

Number of charge carriers per unit volume: $n = p = \frac{\varphi\eta\tau}{wdl}$

Hence, the resistance is:

$$R_d = \frac{1}{\sigma}\frac{l}{wd} = \frac{1}{q\left(\mu_n n + \mu_p p\right)}\frac{l}{wd} = \frac{1}{q\left(\mu_n + \mu_p\right)}\frac{wdl}{\varphi\eta\tau}\frac{l}{wd} = \frac{l^2}{q\left(\mu_n + \mu_p\right)\varphi\eta\tau}$$

# Photoconductive Gain, Quantum Efficiency & Responsivity

# The Photoconductive Gain (1)

(1) Time for an e⁻ to drift from one electrode to the other: $\tau_t = -\frac{l}{\langle v_x \rangle}$

(2) Recall the electron mobility: $\mu = -\frac{\langle v_x \rangle}{E_x}$

Combining (1) and (2) yields: $\tau_t = \frac{l}{\mu E_x}$

→ Define the photoconductive gain: $G \equiv \frac{\tau}{\tau_t} = \frac{\tau \cdot \mu E_x}{l}$

where $\tau$ is the mean carrier lifetime before recombination.

➔ The photoconductive gain is the ratio of carrier lifetime to carrier transit time.

# The Photoconductive Gain (2)

*G quantifies the probability that a generated charge carrier will traverse the extent of the detector <u>and</u> reach an electrode.*

The observed/detected photo current gets degraded by a factor:

$$G = \frac{\tau}{\tau_t}$$

$G << 1$ ⇔ $\tau_t >> \tau$ ⇔ charge carriers recombine before reaching an electrode

$G \sim 1$ ⇔ $\tau_t \sim \tau$ ⇔ all charge carriers are likely to reach an electrode

$G > 1$ is possible if charge multiplication occurs.

# The Photoconductive Gain (3)

How can we optimize the gain *G*?

- make detector as this as possible
- increase the bias voltage ($E_x$)
- eliminate defects and impurities

But optimizing *G* is not sufficient, as the quantum efficiency $\eta$ plays an important role, too.

The product $\eta G$ describes the probability that an incoming photon will produce an electric charge that will penetrate to an electrode.

# Quantum Efficiency

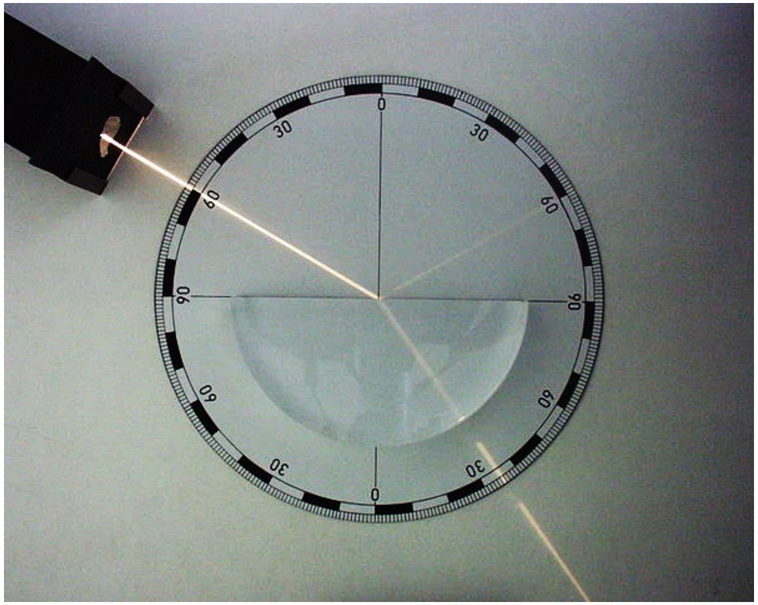
The quantum efficiency $\eta$ is the percentage of photons hitting the detector surface that will produce an electron–hole pair.

Clear definition but difficult to measure.

$\eta$ can be reduced by…:

1. reflection losses at the surface,

$$R = \frac{(n-1)^2}{(n+1)^2} \qquad R_{Ge} = \frac{(4-1)^2}{(4+1)^2} = \frac{9}{25} = 36\%$$




2. loss of photons that cross the detector material without interaction.

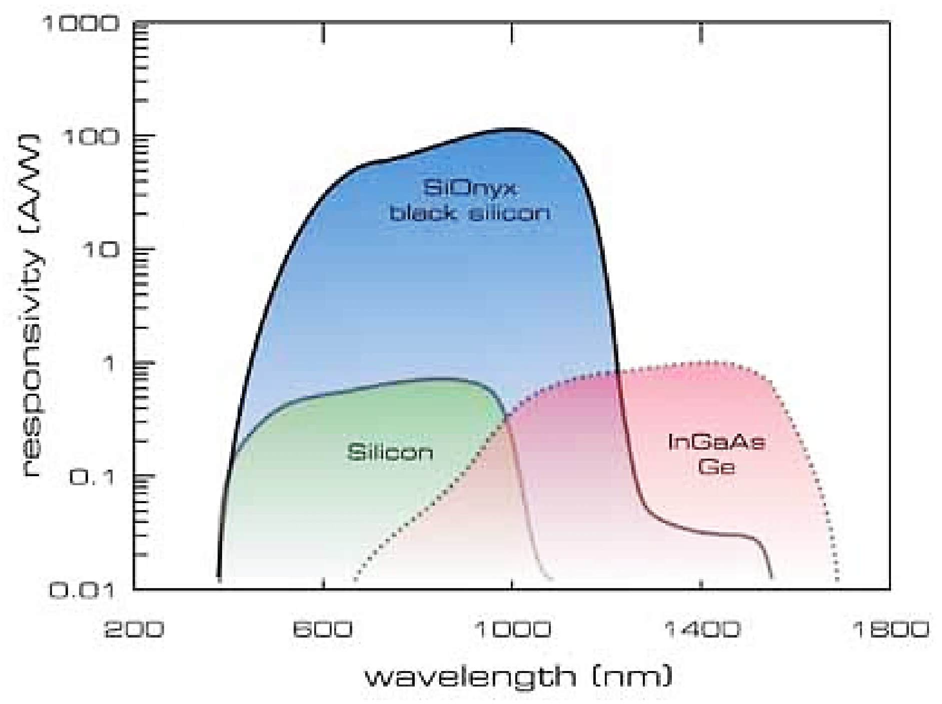
# Responsivity

The responsivity $S$ is the ratio between electrical signal at the detector output and incoming photon power.

Less elegant definition, but easy accessible by measurement.

$$S = \frac{\text{electrical output signal}}{\text{input photon power}}$$

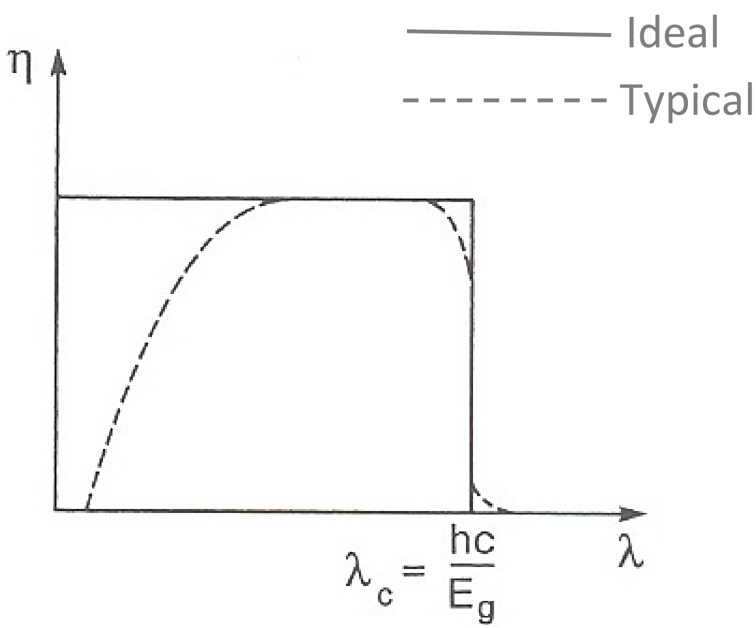
$$S = G\frac{q\eta\lambda}{hc}$$

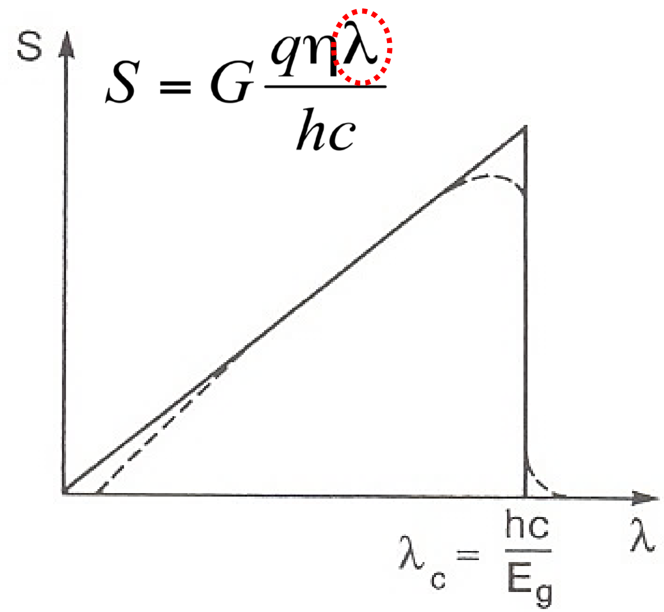


# Quantum Efficiency ⇔ Responsivity

The quantum efficiency $\eta$ is *independent of wavelength* up to the cutoff at $\lambda_c$:

The responsivity $S$ increases proportionally to the wavelength:



# Deriving the “Photo-Current” (3)

# Conductivity σ ⇔ Photon Flux

Responsivity $S \equiv \dfrac{\text{electrical output signal}}{\text{input photon power}}$

The "photon power" falling on the detector is: $P_{ph} = \varphi h \nu = \dfrac{\varphi h c}{\lambda}$

Photoconductive Gain $G \equiv \dfrac{\tau \mu E_x}{l}$

$\tau \cdot \mu / l$ = lifetime × mobility / pathlength

$E_x$ = "amplifying" electric field

The responsivity $S$ then becomes:

$$S = \frac{I_{ph}}{P_{ph}} \overset{Ohm}{=} \frac{V_b}{R_{ph} P_{ph}} \overset{E=V/l}{=} \frac{E_x l}{R_{ph} P_{ph}} = \frac{E_x l}{l^2} q \varphi \eta \tau \left(\mu_n + \mu_p\right) \frac{\lambda}{\varphi h c} = G \frac{q \eta \lambda}{h c}$$

This yields the photo current: $I_{ph} = \dfrac{\eta \lambda q G}{h c} P_{ph} = \eta q \varphi G$

$$R_d = \frac{l^2}{q\left(\mu_n + \mu_p\right)\varphi \eta \tau}$$

# Refresher:

# Noise Distributions

# Gaussian Distribution (1)

Gaussian noise is the noise following a Gaussian (normal) distribution:

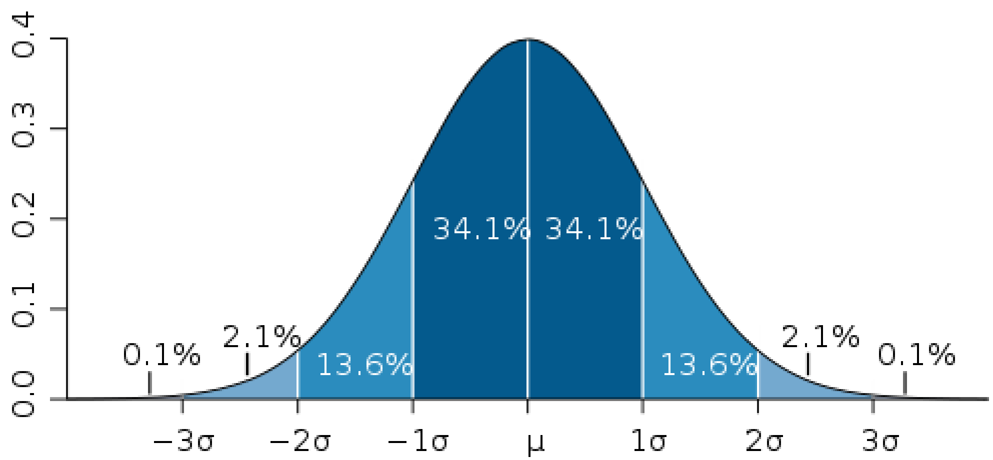
$$S = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[ -\frac{(x-\mu)^2}{2\sigma^2} \right]$$

*It is often (incorrectly) called white noise, which refers to the uncorrelation of the noise.*

$x$ is the actual value

$\mu$ is the mean of the distribution

σ is the standard deviation of the distribution $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$



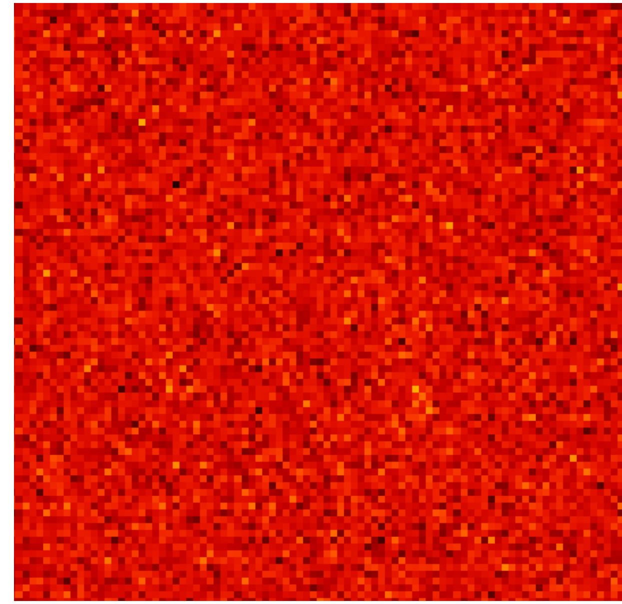

1-σ ~ 68%

2-σ ~ 95%

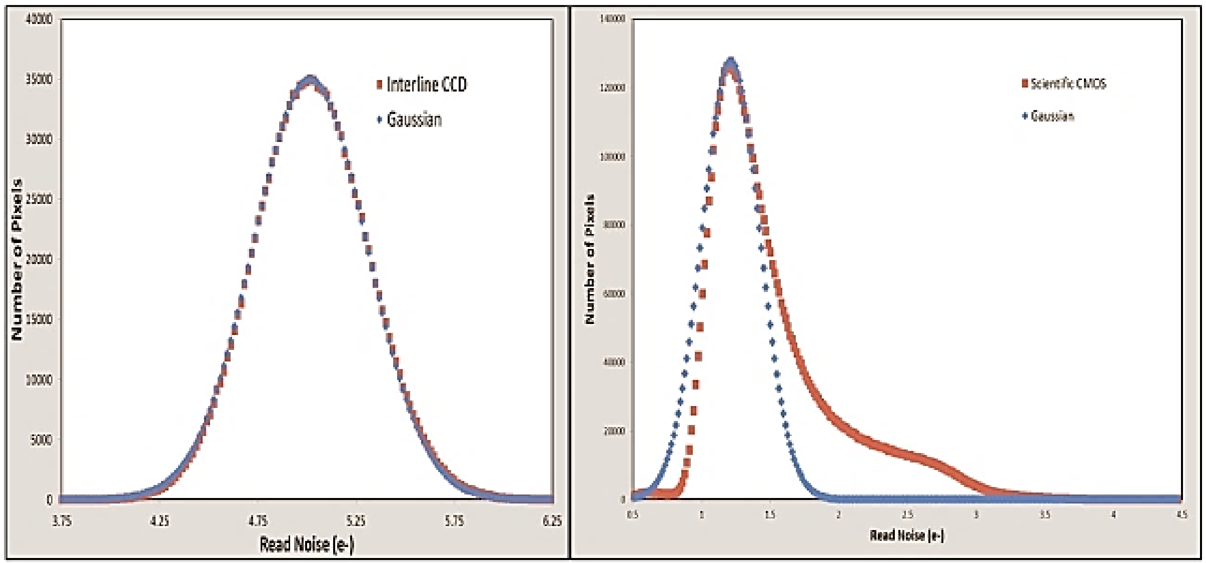
3-σ ~ 99.7%

# Gaussian Distribution (2)

Example: detector "dark frame" (readout without illumination) →



← from: `http://www.microscopy-analysis.com/editorials/editorial-listings/digital-camera-technologies-scientific-bio-imaging-part-3-noise-and`




Usually the dark frames show Gaussian behavior – but not always, in case of other, systematic noise sources (dead pixels, warm electrodes, etc. )

# Poissonian Distribution (1)

Poisson noise is the noise following a Poissonian distribution:

$$P(k,\lambda)=\frac{\lambda^k e^{-\lambda}}{k!}$$

*k* is the number of occurrences of an event (probability)
*λ* is the *expected* number of occurrences

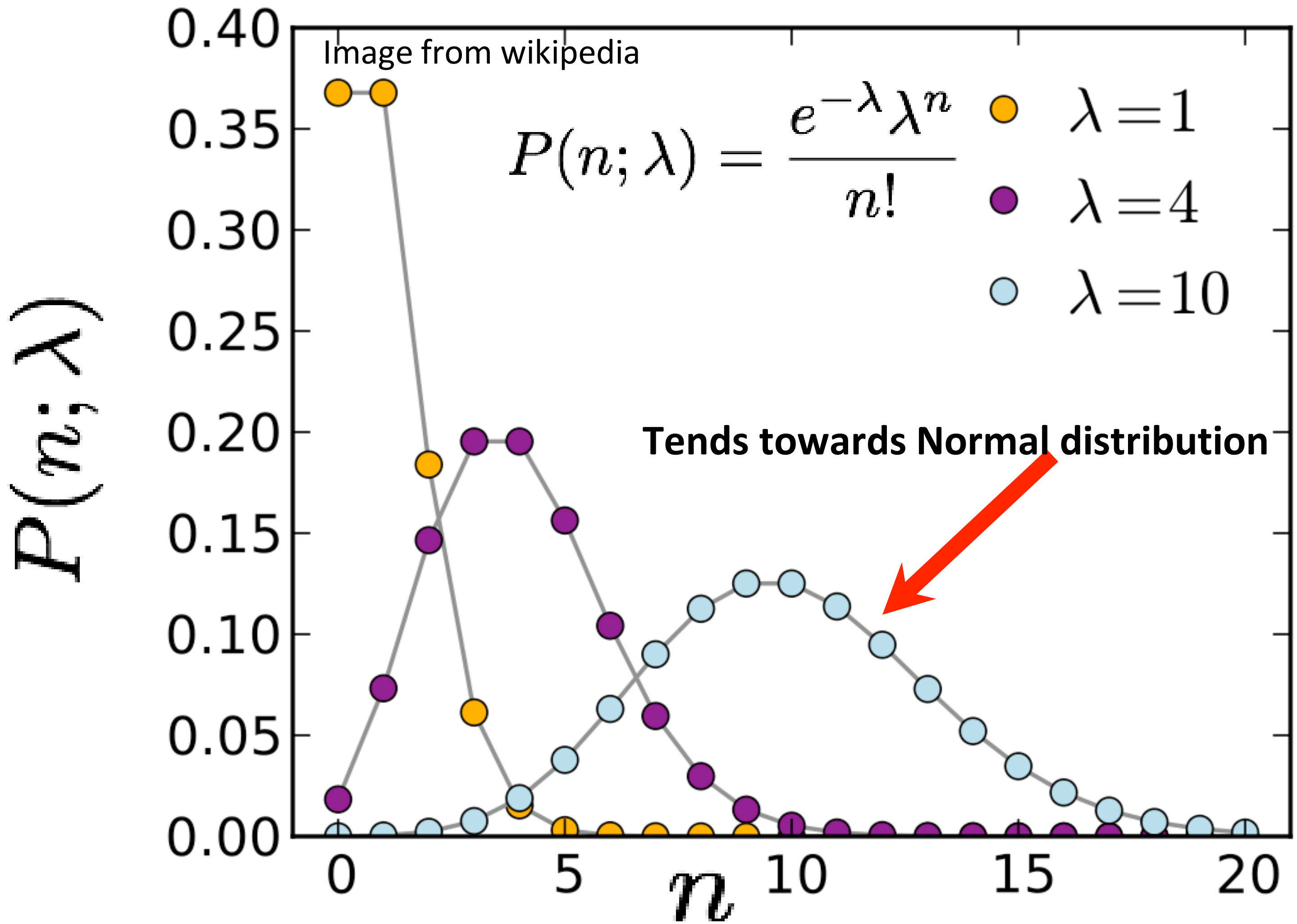
*P(k,λ)* expresses the probability of a number of events occurring in a fixed period of time, provided that:

- these events occur with a known average rate *λ*, and
- the arrival of one event is independent of the time since the last event

Properties:

- the mean (average) of *P(k,λ)* is *λ*.
- the standard deviation of *P(k,λ)* is *√λ*.

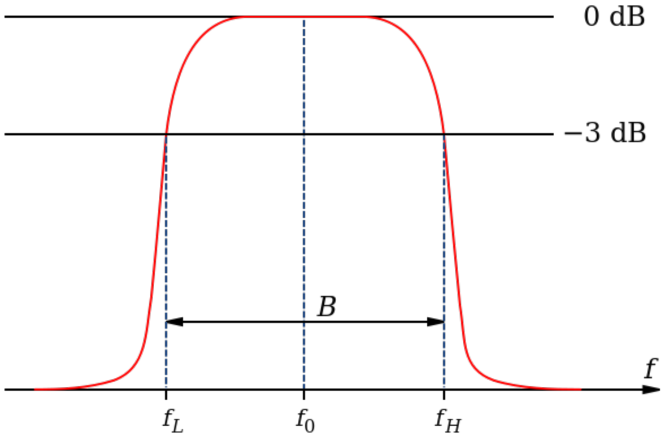
# Poissonian Distribution (2)




Example: fluctuations in the detected photon flux between time intervals $\Delta t_i$.
Detected are $k$ photons, while expected are, on average, $\lambda$ photons.

# Noise Bandwidth

White noise has a wide frequency range, which we associate with an equivalent noise bandwidth $B$ or $\Delta f = f_H - f_L$.




For a system – like our detector – with exponential response $U \sim e^{-t/\tau}$ ,

we get $\Delta f = \dfrac{1}{4\tau}$

*According to the Shannon-Nyquist theorem, an output bandwidth of one hertz is equivalent to half a second of integration.*

→ signal integrated over time $\Delta t_{int}$: $$\Delta f = \frac{1}{2\Delta t_{int}}$$

# The Main Sources of Detector Noise

# The G-R Noise Current

Photoconductor absorbs *N* photons: $N = \eta\varphi\Delta t$

→ create *N* conduction electrons and *N* holes (but consider only e⁻ since $\mu_{e-} \gg \mu_p$)

Randomly **g**enerated e⁻ ***and*** randomly **r**ecombined e⁻ → two random processes

Hence: RMS noise ~ $(2N)^{1/2}$.

Now calculate the associated noise current: $\left\langle I_{G-R}^2 \right\rangle^{1/2} = \frac{q\sqrt{2N}G}{\Delta t}$

With the mean photo-current $I_{ph} = \eta q \varphi G$

one gets: $\left\langle I_{G-R}^2 \right\rangle = \frac{q^2(2N)G^2}{(\Delta t)^2} = \left(\frac{2q}{\Delta t}\right)\left(\frac{qNG}{\Delta t}\right)G = \left(\frac{2q}{\Delta t}\right)\left\langle I_{ph}\right\rangle G$

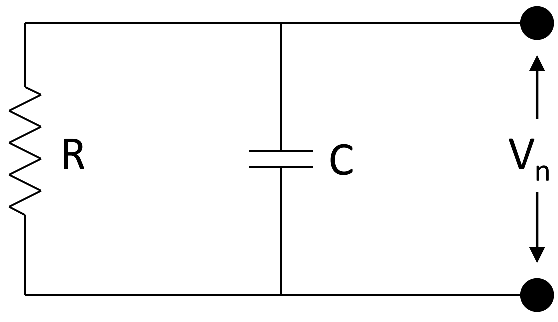
The noise current $<I^2_{G-R}>$ can now be rewritten as:

$$\left\langle I_{G-R}^2 \right\rangle = \left(\frac{2q}{\Delta t}\right)\left\langle I_{ph}\right\rangle G = (2q2\Delta f)\langle \varphi q \eta G\rangle G = \boxed{4q^2\varphi\eta G^2\Delta f}$$

$\Delta f = \frac{1}{2\Delta t_{\mathrm{int}}}$ $\quad I_{ph} = \frac{\eta\lambda q G}{hc}P_{ph} = \eta q\varphi G$

# Johnson (or kTC) Noise (1)

Consider a detector pixel as an RC circuit:




The energy stored in the capacitor is $E_{st} = ½CV^2$.

This system has *one* degree of freedom: $V_n$. Fluctuations in $V_n$ are associated with an average energy of $½kT$:

$$\frac{1}{2}C\left\langle V_n^2 \right\rangle = \frac{1}{2}kT$$

These fluctuations in $E_{st}$ result in a Johnson noise current $I_J$.

The charge on the capacitor is $Q = CV$ → $\langle Q^2 \rangle = C^2V^2 = kTC$

Hence, this noise is also called *kTC* noise or reset noise.

# Johnson (or kTC) Noise (2)

The power in $I_J$ can thermodynamically also be expressed as:

$$\langle P \rangle t = \frac{1}{2} kT \quad \text{with time constant} \quad \tau = t = RC$$

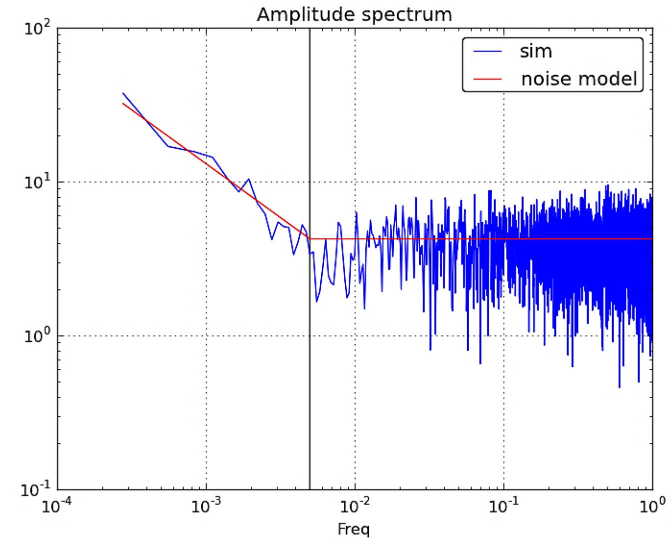
$$\text{and} \;\; P = U \cdot I = R \cdot I^2 \quad \xrightarrow{1\,\text{dof}} \quad \langle P \rangle = \frac{1}{2} \langle I^2 \rangle R$$

Hence,
$$\langle I_J^2 \rangle = \frac{2\langle P \rangle}{R} = \frac{2\frac{1}{2}kT}{Rt} \overset{\Delta f = 1/4\tau}{=} \frac{4kT}{R} \Delta f$$

The Johnson (or *kTC*) noise is a fundamental thermodynamic noise, due to the thermal (Brownian) motion of the charge carriers.

# 1/*f* Noise

Most electronic devices have increased noise at low frequencies, often dominating the system performance.




*However, there is no general physical understanding of it.*

Empirically, $\left\langle I_{1/f}^2 \right\rangle \propto \frac{I^a}{f^b} \Delta f \quad \text{where} \quad a \approx 2, b \approx 1.$

*May be caused by bad electrical contacts, temperature fluctuations, surface effects (damage), crystal defects, and junction field effect transistors (JFETs), etc.*

This type of noise is empirically termed 1/*f* noise.

# Noise Sources in Comparison

# Total Noise

the G-R or noise current $\langle I_{G-R}^2 \rangle = 4q^2 \varphi \eta G^2 \Delta f$

the Johnson noise $\langle I_J^2 \rangle = \frac{4kT}{R} \Delta f$

the 1/*f* noise $\langle I_{1/f}^2 \rangle \propto \frac{I^a}{f^b} \Delta f$

Note that all processes depend on the bandwidth $\Delta f = 1/(2\Delta t_{int})$

If the signal is Poisson distributed in time, the relative error of the measurement is proportional to $1/\sqrt{t}$ or $(\Delta f)^{1/2}$ (longer $t_{int}$ means smaller bandwidth, which means smaller relative errors)

The total noise in the system is $\langle I_N^2 \rangle = \langle I_{G-R}^2 \rangle + \langle I_J^2 \rangle + \langle I_{1/f}^2 \rangle$

# Background-limited Performance

*Ideally, we want the detector sensitivity not being limited by technical factors, but by processes in nature (i.e., nothing we can do about).*

That implies: $\left\langle I_{G-R}^2 \right\rangle >> \left\langle I_J^2 \right\rangle + \left\langle I_{1/f}^2 \right\rangle$

This condition is called background-limited performance **(BLIP)**

BLIP has significant impact on...:

- Detector design: e.g., MIR detectors for ground or space? (→ $I_{dark}$)
- Instrument design: e.g., pixel scale (→ oversampling)
- Observation planning: e.g., exposure time (→ long or short?)

# Noise Equivalent Flux Density (NEFD)

The noise equivalent flux density (NEFD) is the flux density that yields an RMS S/N of unity in a system of $\Delta f$ = 1 Hz.

$$NEFD = \frac{E_S \sqrt{2 \Delta t_{\mathrm{int}}}}{S/N}$$

...where $E_S$ [W m$^{-2}$ Hz$^{-1}$] is the measured flux density.

*The NEFD usually refers to the entire system performance, including the camera optics.*

# Noise Equivalent Power (NEP)

The noise equivalent power (NEP) is the signal power that yields an RMS S/N of unity in a system of $\Delta f$ = 1 Hz.

$$df = \frac{1}{2\Delta t_{int}}$$

$$\frac{S}{N} = \frac{P_s}{\mathrm{NEP}(df)^{1/2}} = \frac{P_s(2\Delta t_{int})^{1/2}}{\mathrm{NEP}} \rightarrow NEP = \frac{P_S\sqrt{2\Delta t_{\mathrm{int}}}}{S/N}$$

An equivalent, more practical definition is:

$$NEP = \frac{I_N}{S}$$

...where $I_N$ [W Hz$^{-1/2}$] is the total noise current in the system, and $S$ [A W$^{-1}$] is the responsivity.

# NEP for BLIP ⇔ kTC

(1) BLIP: $\left\langle I_{G-R}^2 \right\rangle >> \left\langle I_J^2 \right\rangle + \left\langle I_{1/f}^2 \right\rangle$

With $S = G\frac{q\eta\lambda}{hc}$ and $\langle I_{G-R}^2 \rangle = 4q^2\varphi\eta G^2\Delta f$ one gets:

$$\mathrm{NEP}_{G-R} = \frac{I_{G-R}}{S} = \frac{(4q^2\varphi\eta G^2)^{1/2}hc}{Gq\eta\lambda} = \frac{2hc}{\lambda}\left(\frac{\varphi}{\eta}\right)^{1/2}$$

(The factor of $\Delta f$ disappears from $<I^2_{G-R}>$ as we use a "normalized" noise current in units of [A Hz$^{-1}$].)

(2) kTC: $\left\langle I_J^2 \right\rangle >> \left\langle I_{G-R}^2 \right\rangle + \left\langle I_{1/f}^2 \right\rangle$

With $S = G\frac{q\eta\lambda}{hc}$ and $\langle I_J^2 \rangle = \frac{4kT\Delta f}{R}$ one gets:

$$\mathrm{NEP}_J = \frac{I_J}{S} = \frac{(4kT)^{1/2}hc}{R^{1/2}Gq\eta\lambda} = \frac{2hc}{Gq\eta\lambda}\left(\frac{kT}{R}\right)^{1/2}$$

Here, the NEP can be improved by increasing *η*, *G*, *R* or reducing *T*.