Direct detection of exoplanets using long-baseline interferometry and visibility phase

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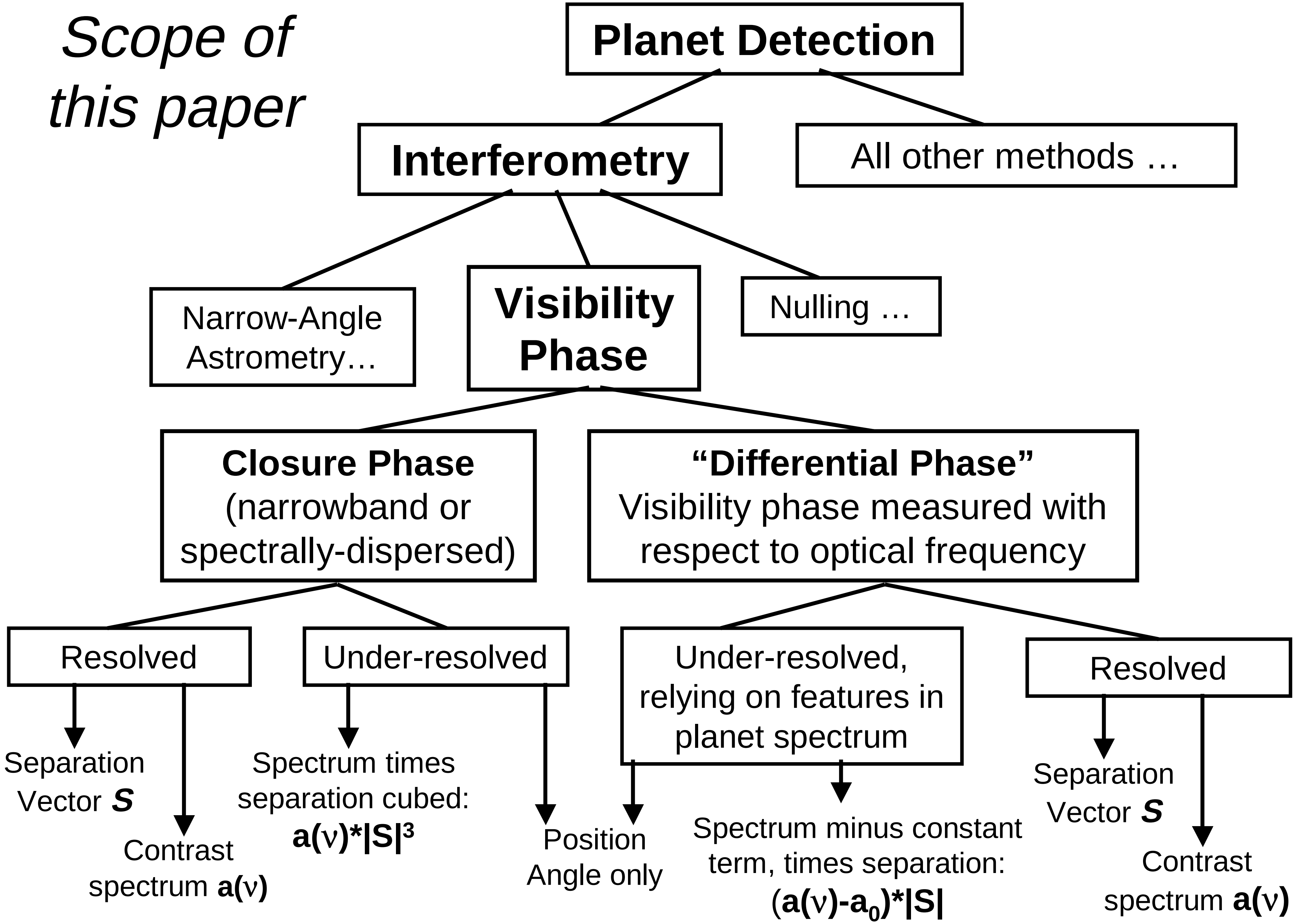
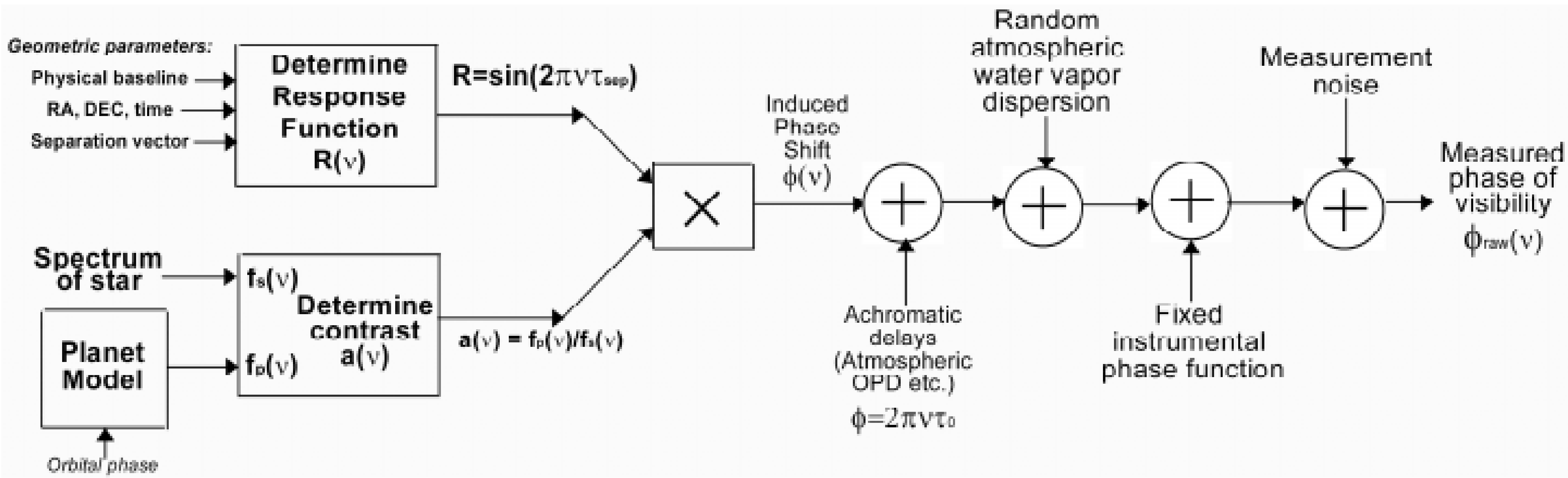

Scope of this paper
Planet Detection
Interferometry
All other methods …
Narrow-Angle Astrometry…
Visibility Phase
Nulling …
Closure Phase
(narrowband or spectrally-dispersed)
"Differential Phase"
Visibility phase measured with respect to optical frequency
Resolved
Under-resolved
Under-resolved, relying on features in planet spectrum
Resolved
Separation Vector S
Contrast spectrum a(ν)
Spectrum times separation cubed:
a(ν)*|S|³
Position Angle only
Spectrum minus constant term, times separation:
(a(ν)-a0)*|S|
Separation Vector S
Contrast spectrum a(ν)

Creation of visibility phase: Overview



Or in summary:

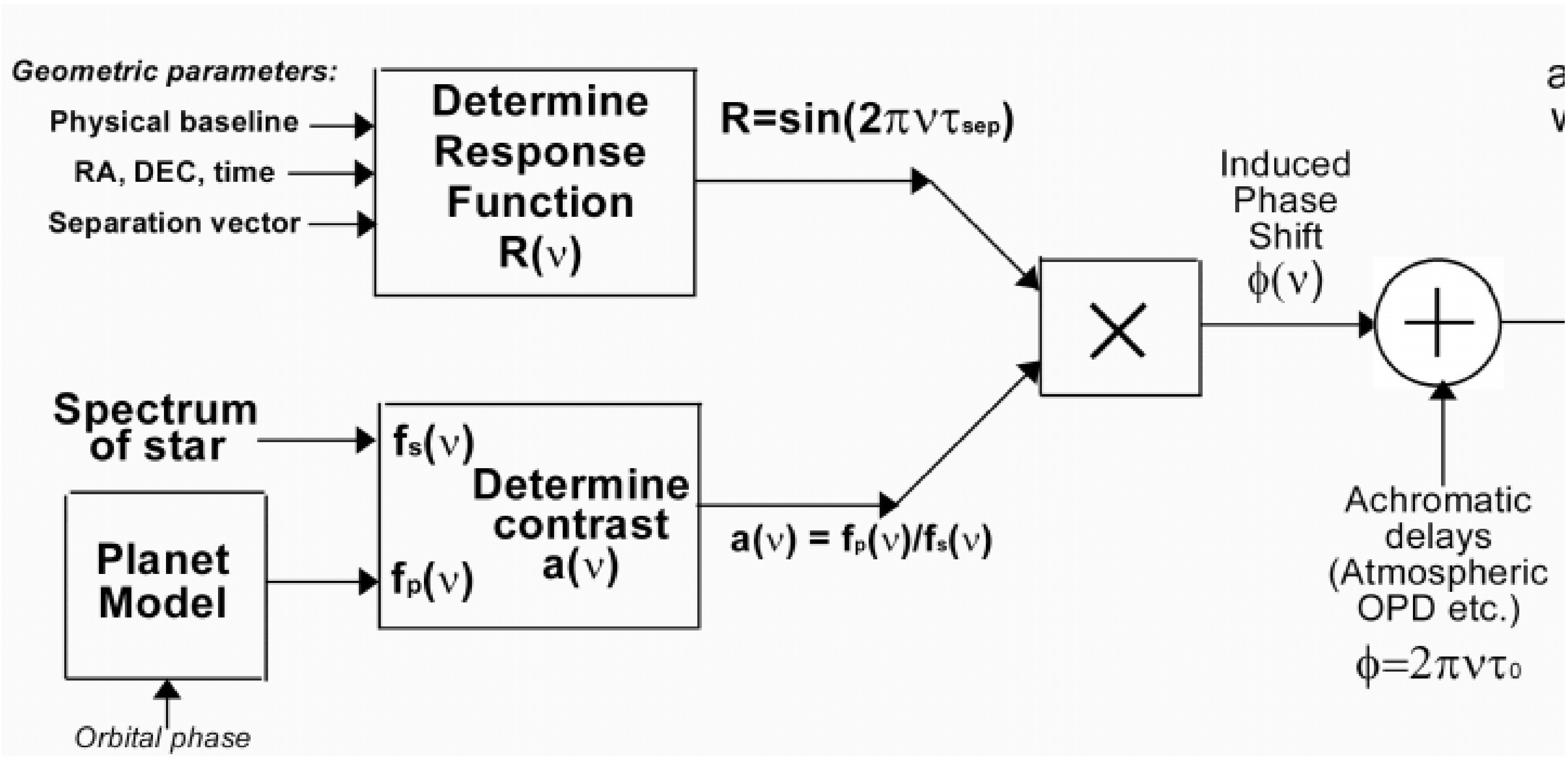
$\phi_{raw} = a(\nu)\, R(\nu) + \phi_{other}$ ← Additional phase effects (undesired!)

Response Function (from geometry) → $R(\nu)$

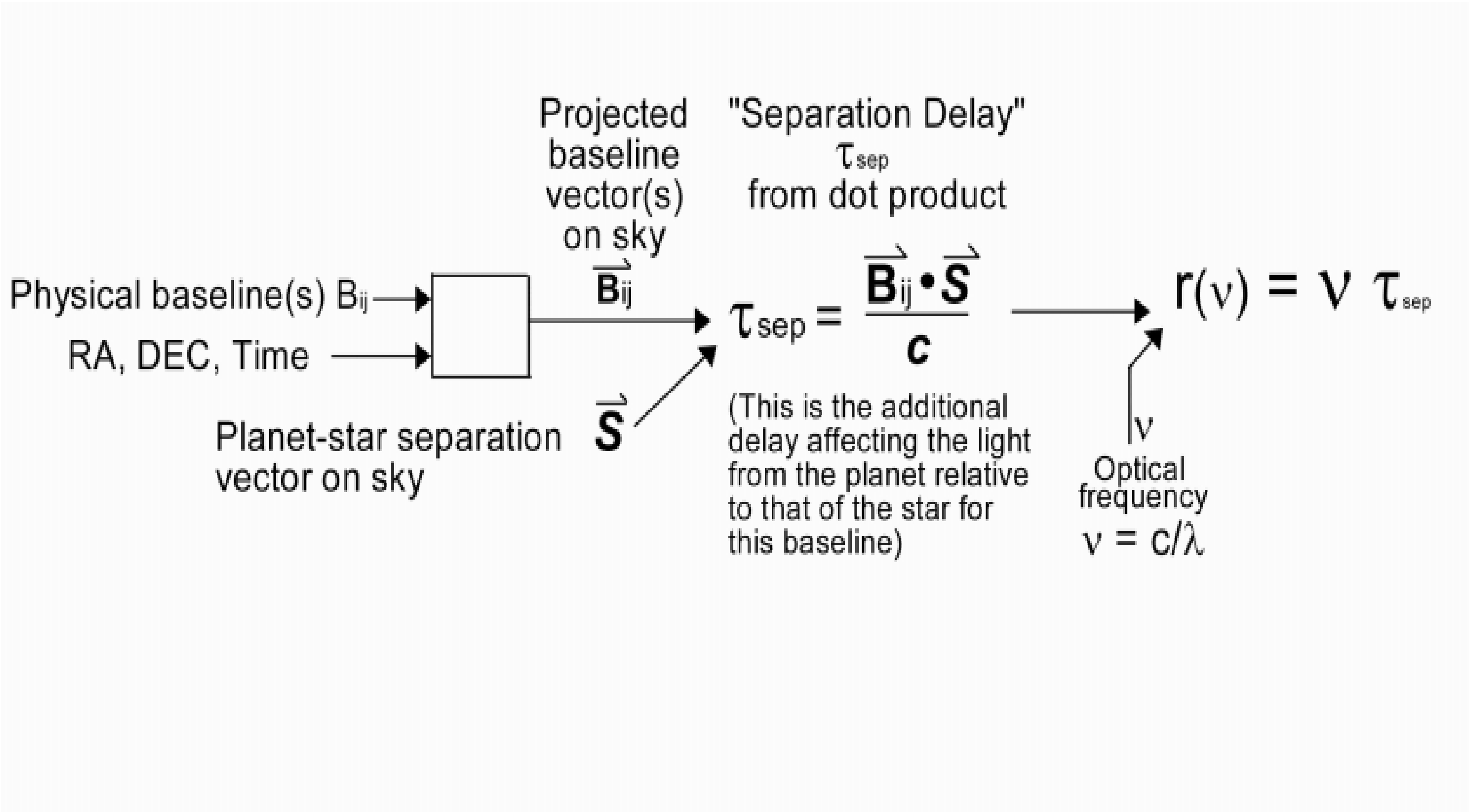
Planet spectrum (relative to star) → $a(\nu)$

Creation of visibility phase:

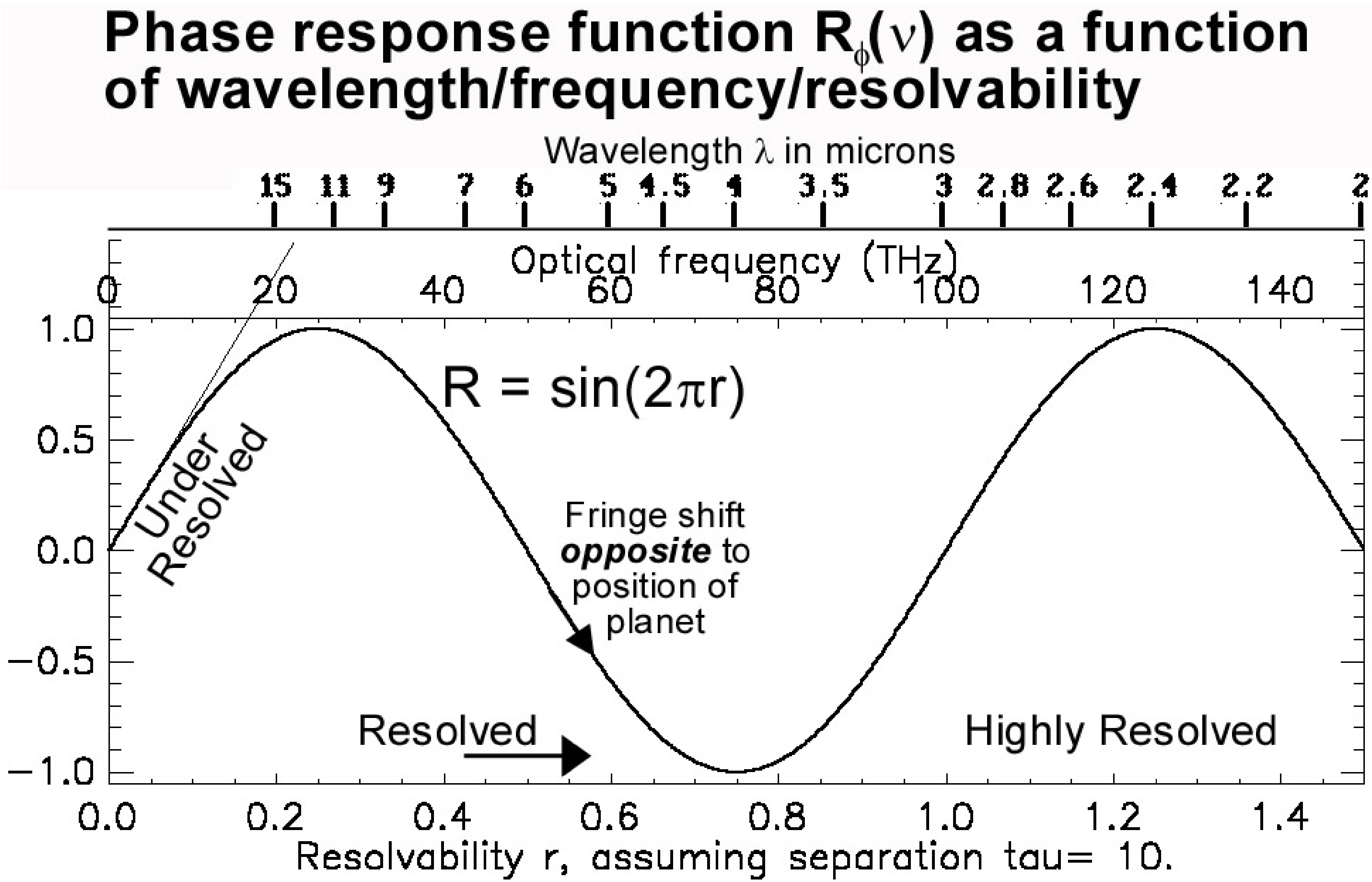
Response function times contrast function



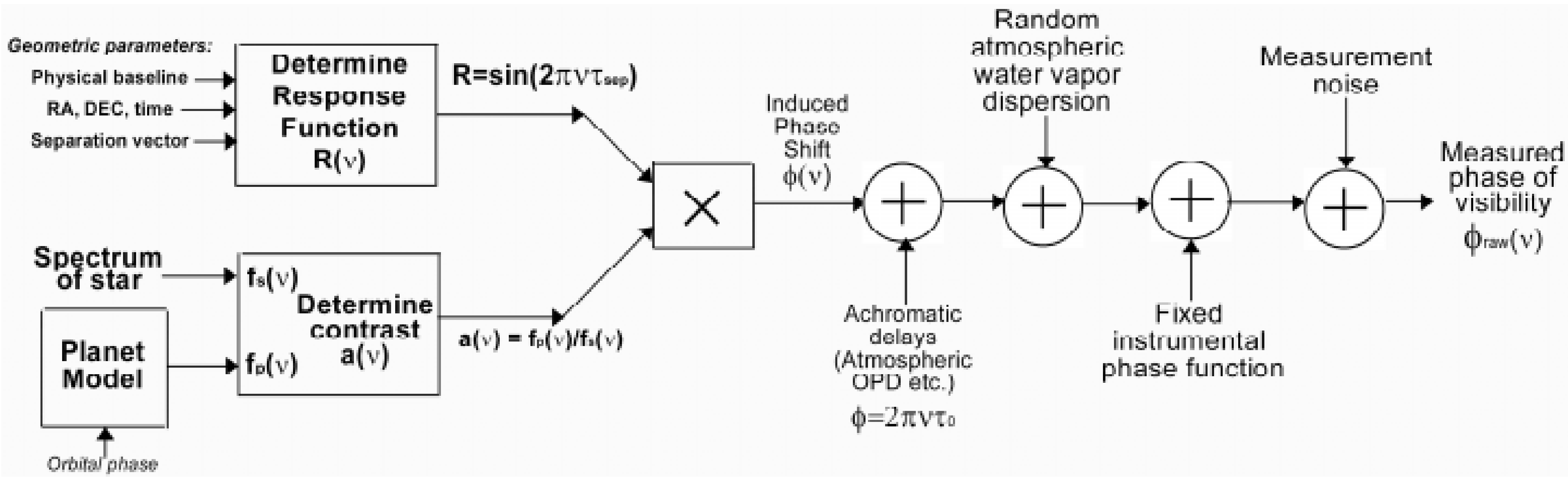
Calculation of “resolvability” **r**



Creation of visibility phase:



Creation of visibility phase:
(Overview)



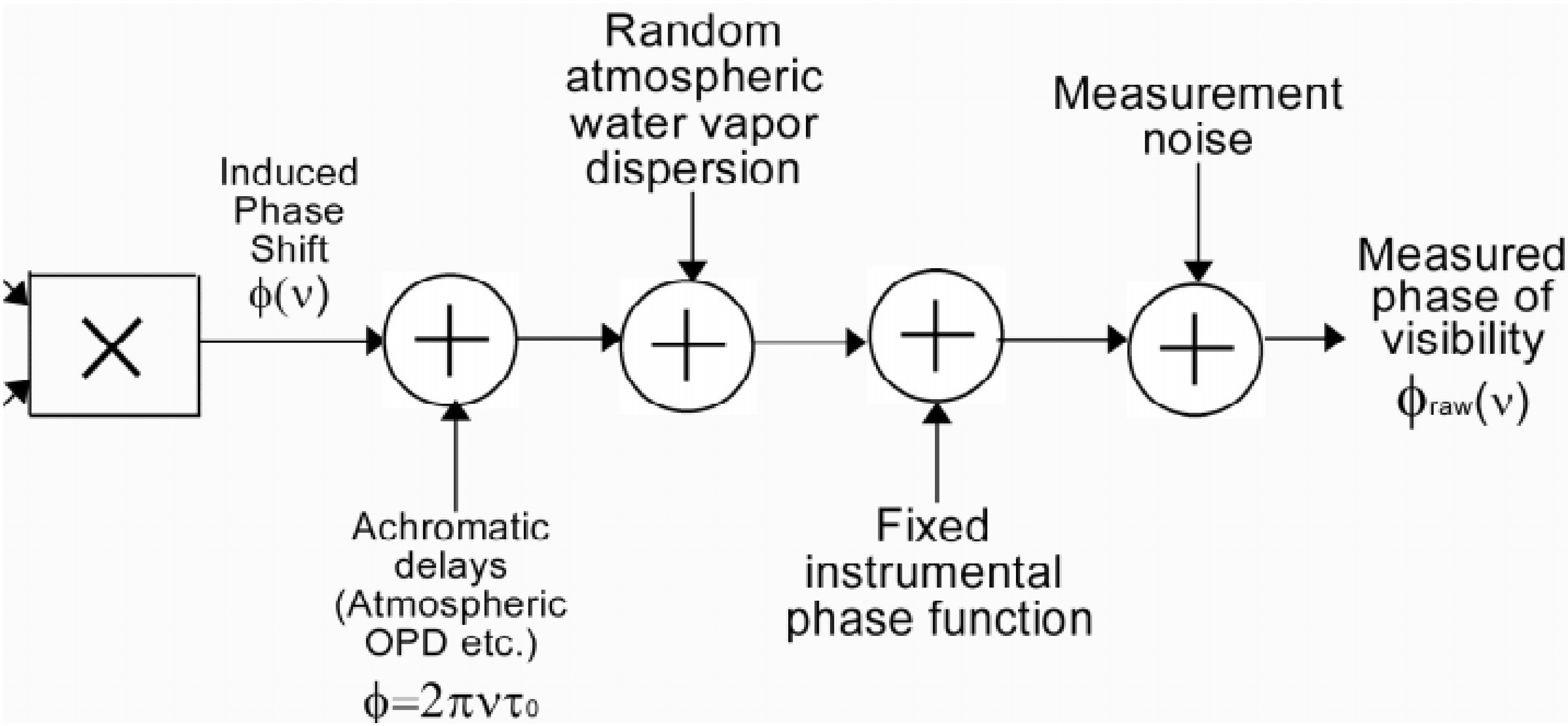
Or in summary:

$\phi_{raw} = a(\nu)\, R(\nu) + \phi_{other}$ ← Additional phase effects (undesired!)

$R(\nu)$ ← Response Function (from geometry)

$a(\nu)$ ← Planet spectrum (relative to star)

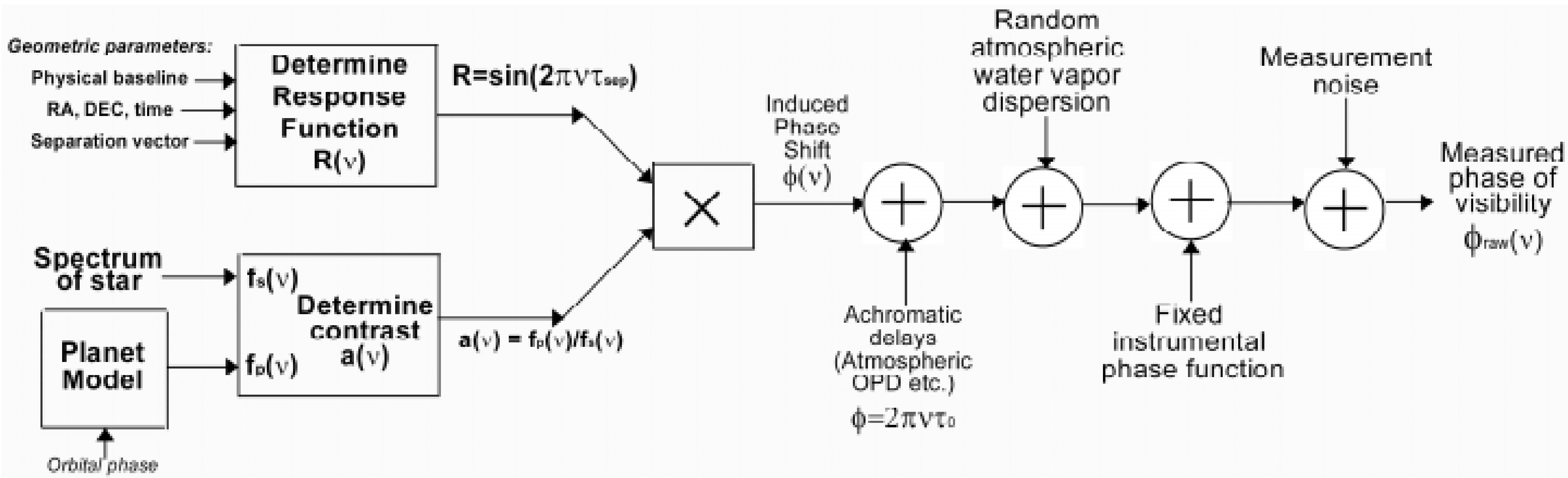
Degradation of visibility phase



Solutions to phase degradation effects

Effect	Using Phase Closure	Using Differential Phase
OPD offset τ_0	*Cancels*	Use phase delay rather than phase, and solve for position of star, τ_s
Atmospheric water-vapor dispersion	*Cancels*	Use “waterless” signal. Allow for arbitrary amount of water vapor in path. No sensitivity to “pseudo-water” in model.
Instrumental dispersion (optical train)	*Cancels*	Calibrate response using observation of symmetric object
Instrumental phase response (beam combiner)	Calibrate response using observation of symmetric object	Calibrate response using observation of symmetric object

Creation of visibility phase:
(Overview)



Or in summary:

$\phi_{raw} = a(\nu)\, R(\nu) + \phi_{other}$ ← Additional phase effects (undesired!)

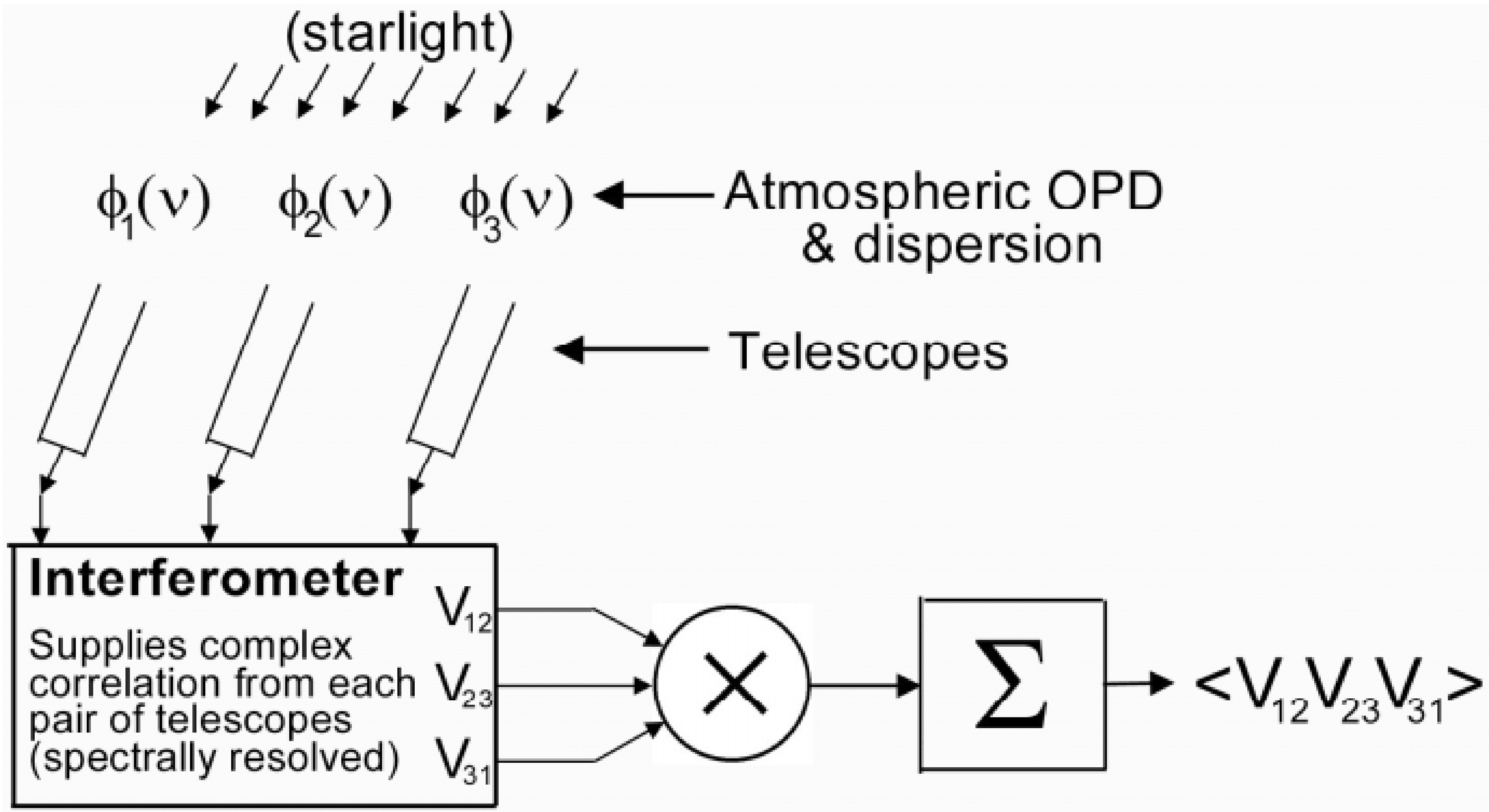
Response Function (from geometry) → $R(\nu)$

Planet spectrum (relative to star) → $a(\nu)$

Approach to estimation of ***S*** and **a**(ν) (resolved observation)

- Initially assume **a**(ν) is constant (or simple black body spectrum)
- Form weighting function on basis of expected signal (using current estimate of ***S*** and **a**(ν)) and detection noise level over ν
- Apply weighting in calculating correlation of data with resolvability model for all possible ***S***
- Using estimate of ***S***, solve for planet spectrum **a**(ν)
- Repeat

Planet detection using the interferometric ***Closure Phase*** ϕ_c



How the closure phase cancels atmospheric and instrumental dispersion and OPD

Assume that the *underlying* visibilities are called v_{12}, v_{23}, and v_{31}. Then at any instant in time, the *observed* visibilities are:

$V_{12} = \exp(j\phi_1) \exp(-j\phi_2)\, v_{12}$
$V_{23} = \exp(j\phi_2) \exp(-j\phi_3)\, v_{23}$
$V_{31} = \exp(j\phi_3) \exp(-j\phi_1)\, v_{31}$

Therefore, the product of the three observed visibilities:

$V_{12} V_{23} V_{31} = v_{12} v_{23} v_{31}$

Thus we have a coherent estimator (phase is preserved!) which is insensitive to all phase shifts induced up to (but not including) the beam combiner! ☺

Closure phase to detect planets

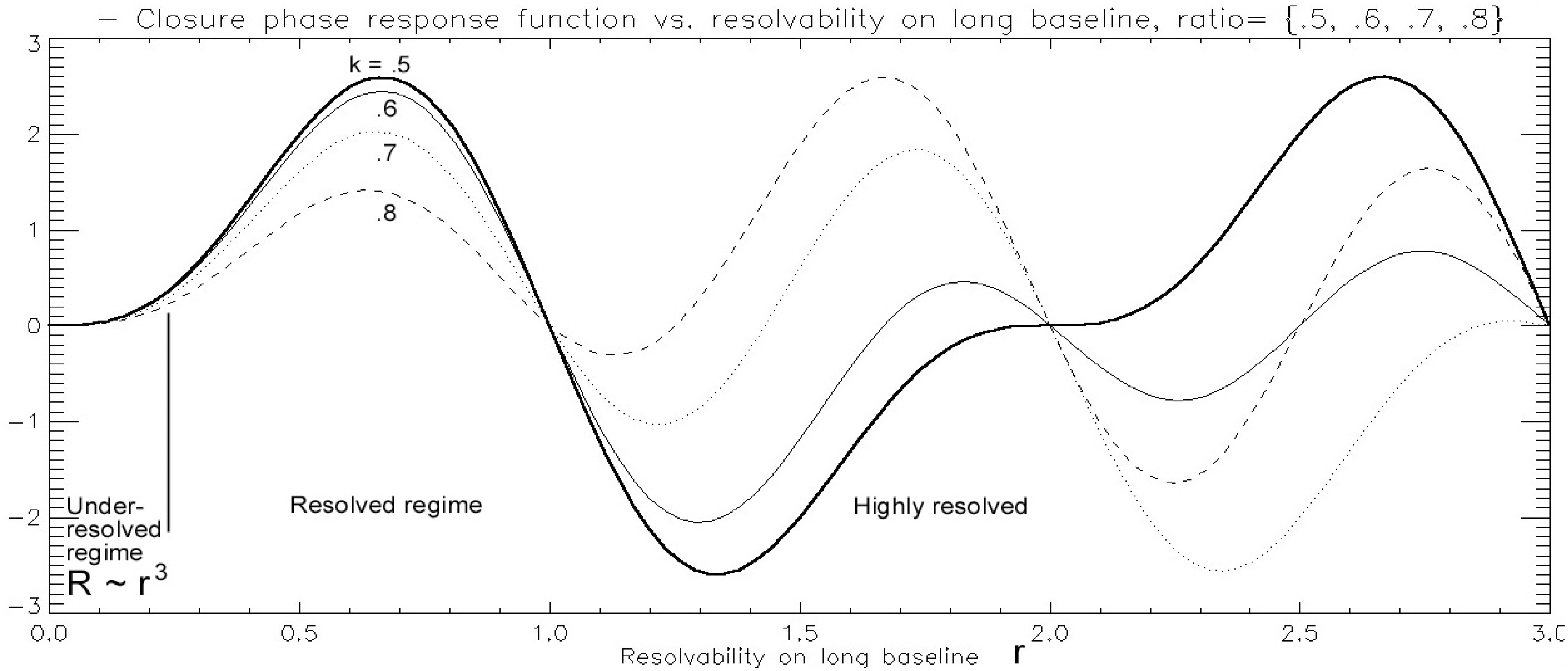
How does closure phase work?

- We take the resolvabilities of the 3 baselines, $\mathbf{r}_{12}$, $\mathbf{r}_{23}$, and $\mathbf{r}_{31}$. Due to the geometry, these necessarily satisfy $\mathbf{r}_{12} + \mathbf{r}_{23} + \mathbf{r}_{31} = 0$
- From these 3 resolvabilities, we can form the closure phase response function $\mathbf{R_c(\nu)}$. This is the ratio of the closure phase generated ϕ_c to the contrast ratio $\mathbf{a(\nu)}$.
- We find for $\mathbf{R_c(\nu)}$:

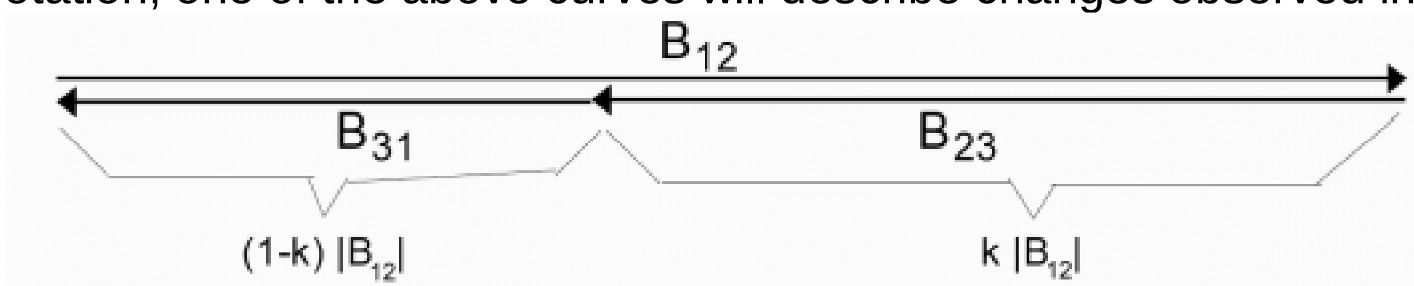
  $$R_c(\nu) = \sin(2\pi\, r_{12}) + \sin(2\pi\, r_{23}) + \sin(2\pi\, r_{31})$$

- Therefore the closure phase is related to the departure from sin(x) = x and becomes insensitive at small resolvabilities.
- Take $\mathbf{r}_{12}$ as the longest baseline, and call the ratio of one of the others to the long baseline **k**. Then let us examine $\mathbf{R_c(r_{12}, k)}$ which is largely a function of the resolvability of the longest baseline, $\mathbf{r}_{12}$, but also a function of k, especially as it departs from ½.

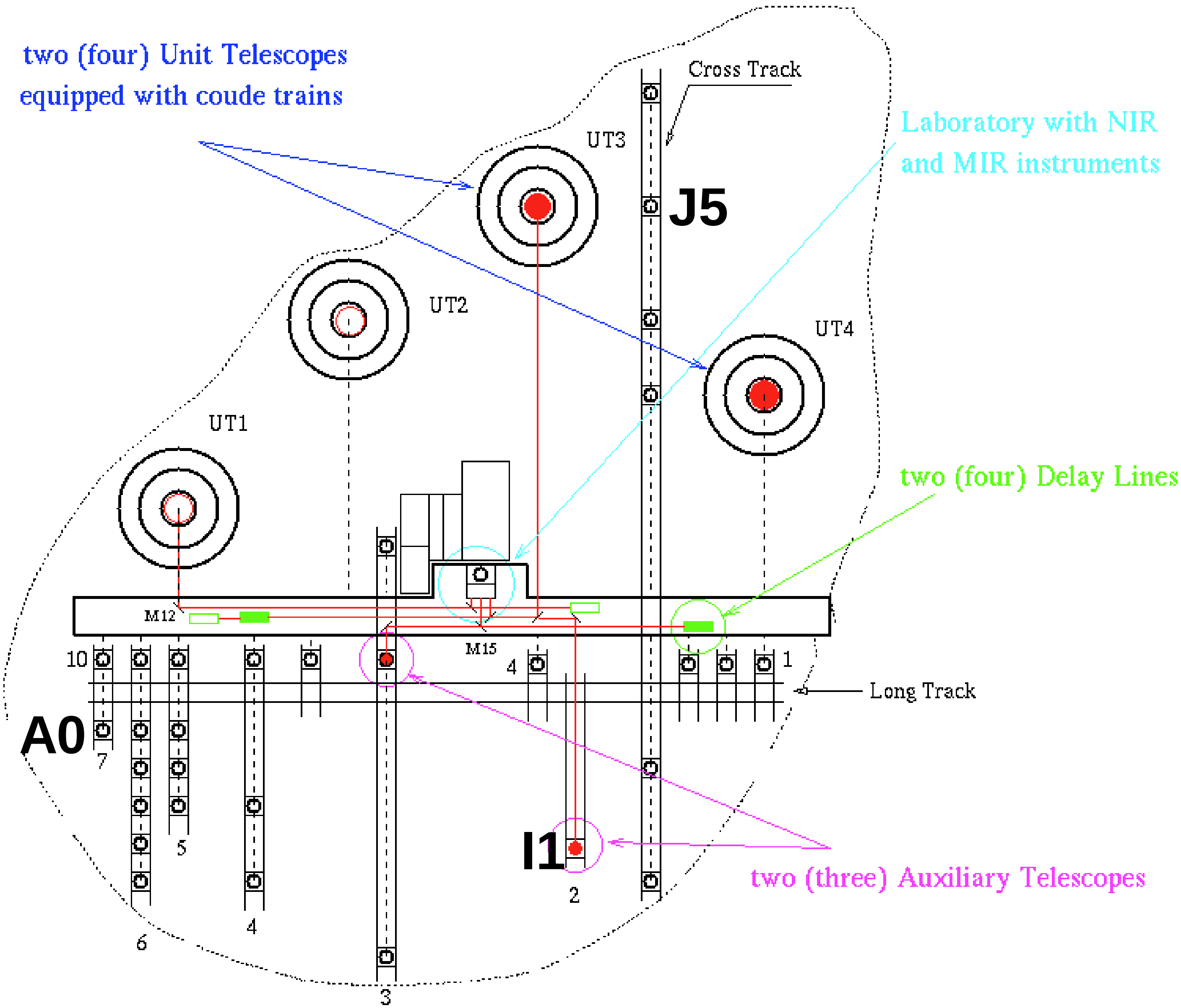
The closure phase response function $R_c(\nu)$



Consider the following special case: a triplet of telescopes arranged along a line. Now **k** is the same for all observations, and as the projected baseline changes with earth rotation, one of the above curves will describe changes observed in ϕ_c

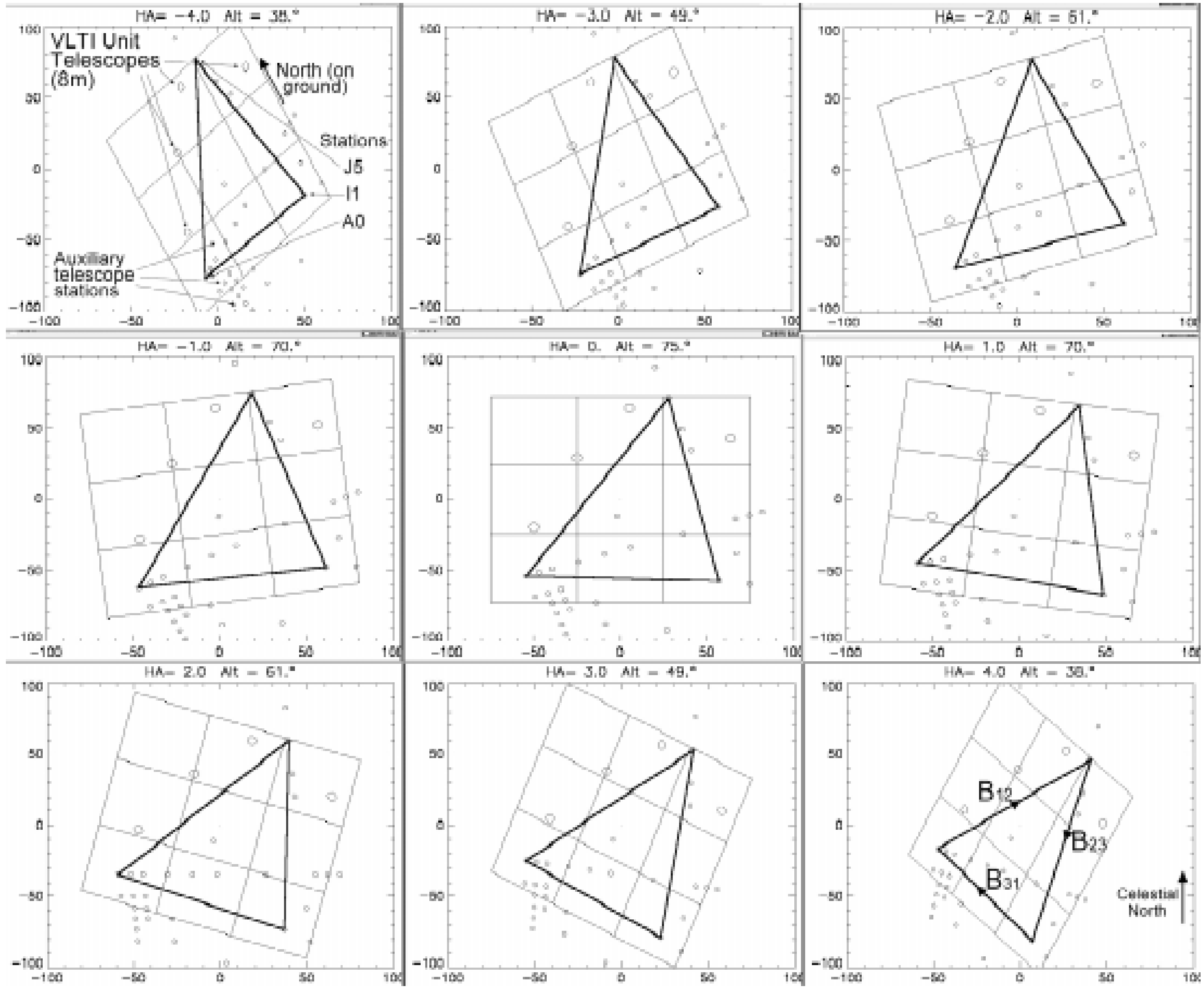


Example of earth rotation synthesis using the baseline configuration of the VLTI



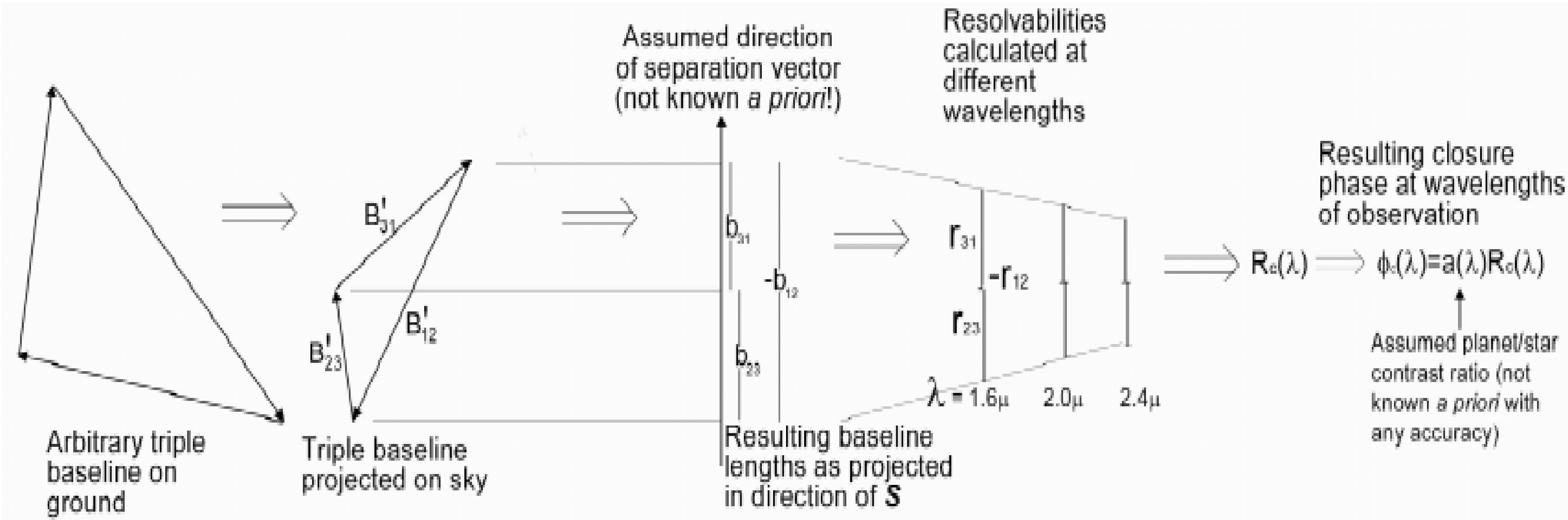
Triple baseline evolution with earth rotation: the general case

It's hard to generalize, but here is an example of the baselines as *projected on the sky*, of the VLTI trio A0 – I1 – J5 over 8 hours (when the star is above 38^o elevation) for a star at declination 40^o S.



Fact: whenever any one baseline is perpendicular to the separation vector ***S*** then $\mathbf{R_c(\nu)} = 0$

Calculating the closure phase response function $R_c(\nu)$ in the general case

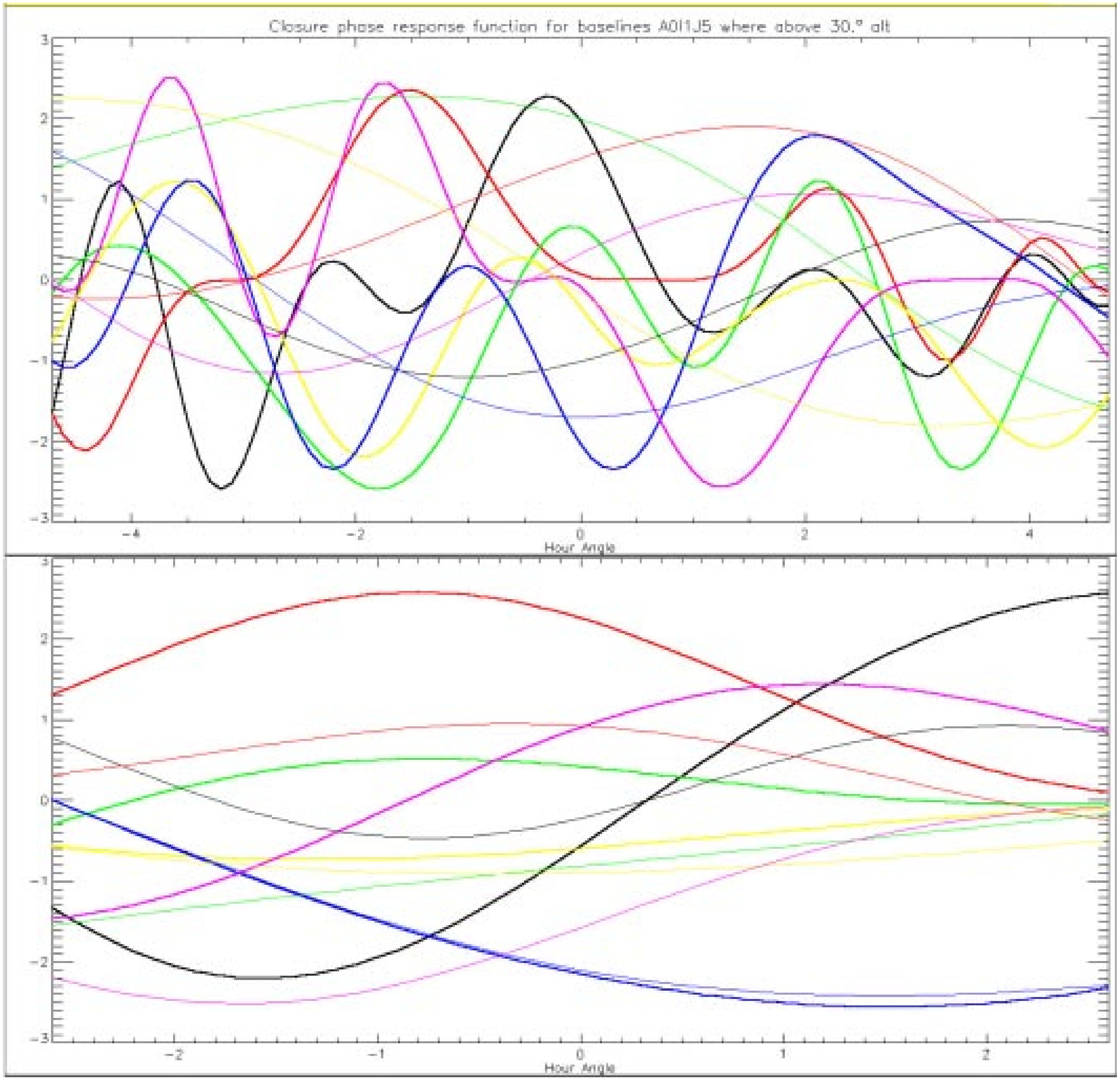


The three baselines as projected in the direction of ***S***, have lengths b_{12}, b_{23}, and b_{31}, where $b_{12} + b_{23} + b_{31} = 0$. Multiplying each by the separation |***S***| and dividing by the wavelength, we can find the resolvabilities: $\mathbf{r}_{ij} = b_{ij}\,|\boldsymbol{S}|\,/\,\lambda$

Response function vs. earth rotation: K band examples using the VLTI

Object at DEC -42^o over 9 hours, using A0 - I1 - J5 baseline trio. Thick plots are for a 10mas separation. Thin plots for 1.65 mas separation (as for HD75289).

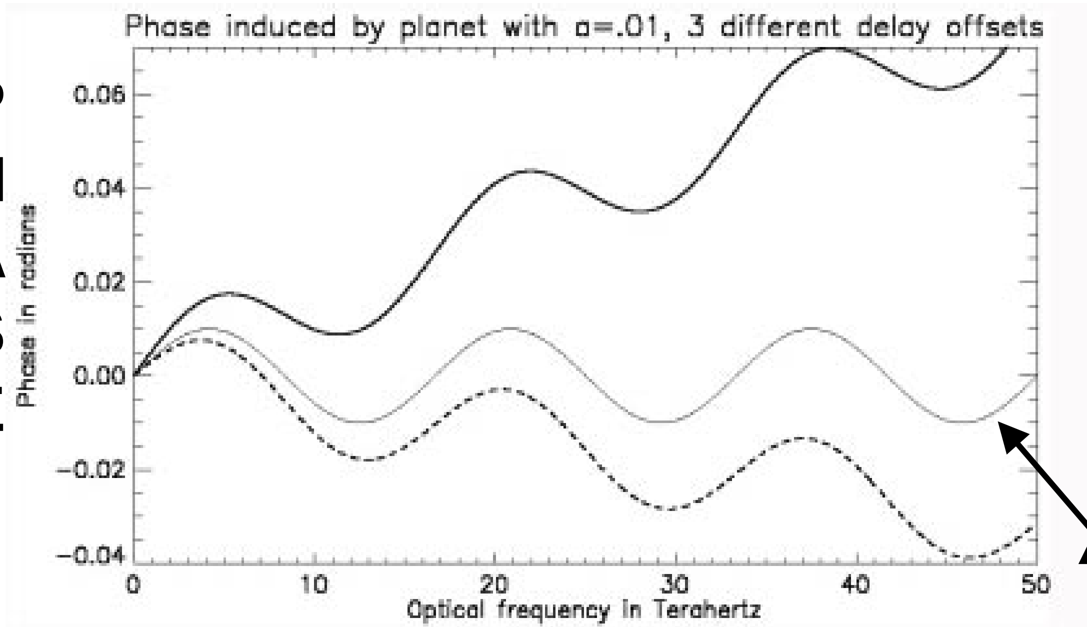
Colors are for each of 6 position angles 30^o apart



Object at DEC +20^o with 3 mas separation (e.g. 51 Peg) over 5 hours, Thick plots using A0 - I1 - J5, thin using A0-M0-G1 baseline trio.

“Differential phase” method

P
H
A
S
E



D
E
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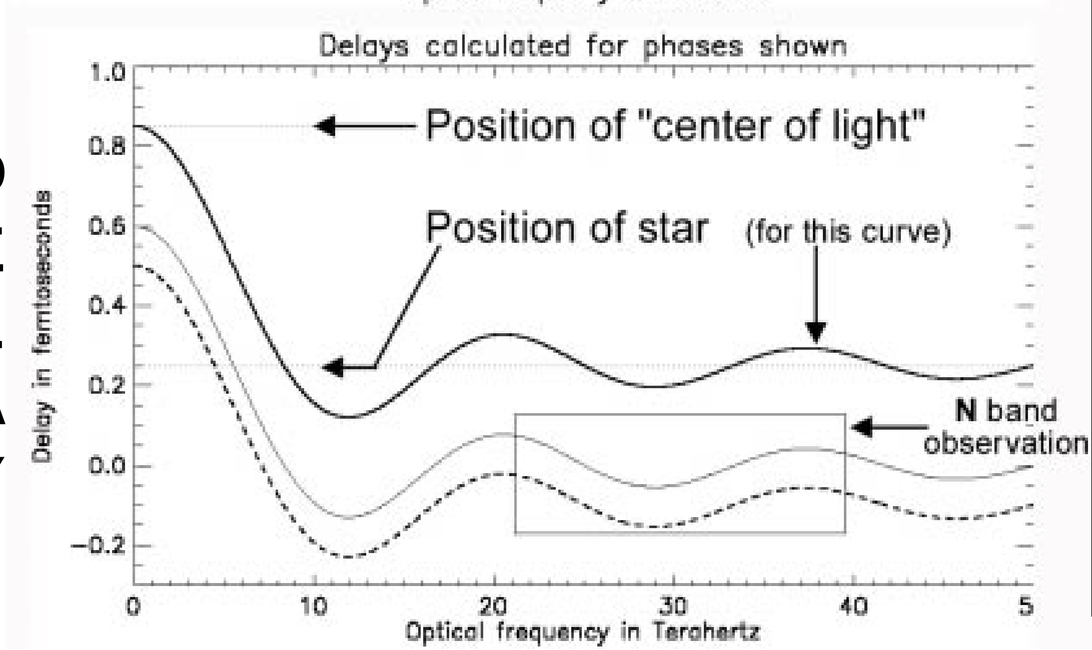


Why you don’t use **phase**, but rather **phase delay.**

Restoration procedure:

1) Obtain one delay curve from the visibility phase
2) Estimate the position of the star, τ_{star} (if possible)
3) In the end, you may convert the phase delay (after subtracting τ_{star}) back into phase: $\phi = 2\pi\nu\tau$ and you will always get this ☺

Differential phase delay method

- Now we form the **delay** response function $\mathbf{R_\tau}$, such that the **phase delay** $\tau(\nu)$ generated by a planet can be factored as:

  $$\tau(\nu) = \mathbf{a}(\nu)\ \mathbf{R_\tau}(\nu) + \tau_{\mathbf{arbitrary}}$$

- In this case we find that $\mathbf{R_\tau(\nu)}$ is given by the sinc function, scaled by the separation delay τ_{sep} :

  $$\mathbf{R_\tau(\nu) = \tau_{sep}\ sinc(2r) = \tau_{sep}\ sinc(2\nu\tau_{sep})}$$

- When r << 1, we find the delay produced is approximately:

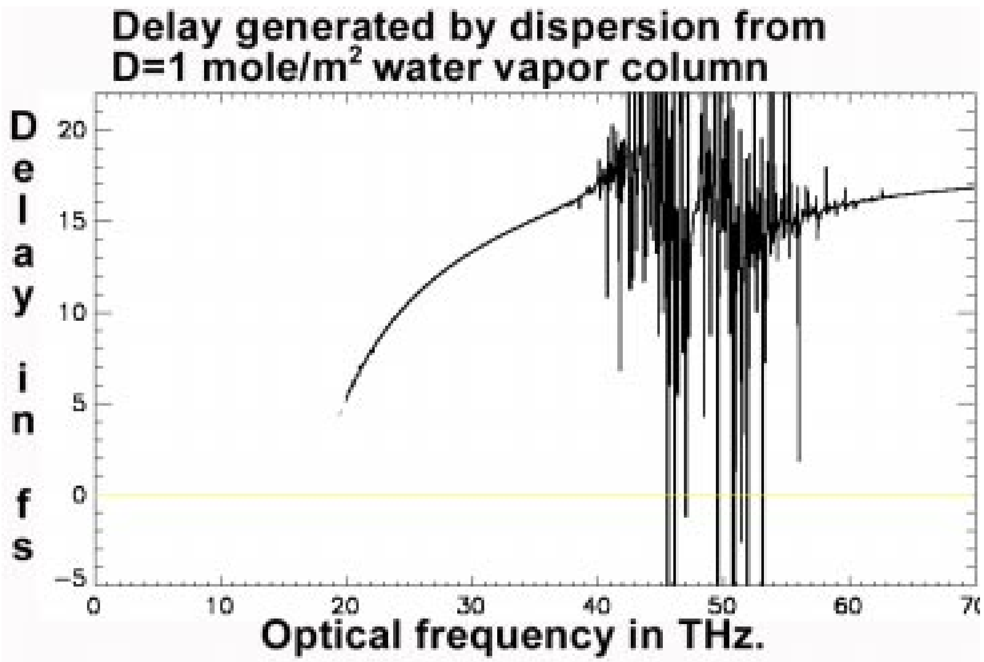
  $\tau(\nu) = \mathbf{a}(\nu)\ \tau_{\mathbf{sep}} = \tau_{\mathbf{CL}}(\nu)$ the so-called “center of light.”

- This is characteristic of an unresolved observation: the star and planet blend into one position (for each wavelength).
- In a resolved observation, the phase delay is closer to the star, and (when r>.5) even moves to the other side of the star. It is that change that makes the planet detectible.

Differential phase delay method

Further complication:

Result contains an arbitrary amount of dispersion from differential atmospheric water vapor transmission



PROBLEM:

- 1 mole/m^2 as shown is a typical level of differential water vapor (D=1) over a typical baseline.
- The process of differential water-vapor fluctuations is zero-mean but is dominated by low-frequencies so that the average over an observation will not be greatly reduced.
- The shown level of phase delay is over 1000 times greater than the expected signature from a planet!

Differential phase delay method

How does the signature of the planet fare when an arbitrary amount of water vapor dispersion is added?

- That depends on the *Differential Resolvability* $\Delta\mathbf{r}$, defined as the change in the resolvability **r** over the frequency limits of the observation.
- For a small $\Delta\mathbf{r}$ <<1, the signal is almost completely degenerate with the added phase delay from dispersion!
- For a large $\Delta\mathbf{r} > 1$, the shape of the planet's differential phase delay, which amounts to a complete cycle of the sinc function, cannot be mimicked by any amount of water vapor dispersion. No loss of SNR occurs.
- For an intermediate $.25 < \Delta\mathbf{r} < 1$, there is a substantial but not overwhelming degradation in the detectibility of the planet.
- Even for an under-resolved observation, $\Delta\mathbf{r}$ <<1, there may be sufficient detectibility in the case of a contrast ratio $\mathbf{a}(\nu)$ which contains prominent spectral features. Detectibility now depends on amplitude and total width of spectral features. These may be due to the star as well as the planet.
- Observations which are under-resolved in an absolute sense, $\mathbf{r}_{\mathbf{max}} < .15$, may determine the position angle of the separation vector ***S***, but only supply an estimate of the offset of the “center of light,” that is, $|\boldsymbol{S}| * \mathbf{a}(\nu)$.

Solving for resolvability **r** (and eventually **a(ν)**) for an observation corrupted by water vapor dispersion

1) Convert the measured phase to delay: $\tau_{raw}(\nu) = \phi_{raw}(\nu) / 2\pi\nu$
2) Determine a **weighting function W(ν)** which is inversely proportional to the measurement noise in the system which estimates $\phi_{raw}(\nu)$.
   Note: the expected value of the eventual result is insensitive to **W**.
3) Form the modified weighting function **W'(ν) = νW(ν)** which applies more weight to delays at higher frequencies where those delays correspond to larger phases.
4) Using the refractivity of water (something proportional to n-1), form a delay function for "clean water" by adjusting its offset. "Clean water" is defined as a substance which has zero average delay using the weighting function W' and can be formed as follows:

   $\boldsymbol{\tau_{H2O}(\nu) = n_{H2O}(\nu) - \tau_0}$ where the offset is given by:

   $\tau_0 = \int d\nu\, \mathbf{W'(\nu)\, n_{H2O}(\nu)}$ (**n_{H2O}** is the refractivity of water)

Solving for resolvability **r** (and eventually **a(ν)**) for an observation corrupted by water vapor dispersion

…. continued:

5) Find the amount of water **w** to subtract, and the amount of achromatic delay τ_{offset} to subtract using the following integrals. (Note: by using "clean water" these two determinations have been orthogonalized).

$$\tau_{offset} = \left(\int d\nu\, W'(\nu)\, \tau_{RAW}(\nu)\right) / \left(\int d\nu\, W'(\nu)\right)$$

$$w = \left(\int d\nu\, W'(\nu)\, \tau_{RAW}(\nu)\, \tau_{H2O}(\nu)\right) / \left(\int d\nu\, W'(\nu)\, (\tau_{H2O}(\nu))^2\right)$$

6) Finally, we form the "waterless" (and offsetless) delay function $\tau_{WL}(\nu)$ as follows:

$$\tau_{WL}(\nu) = \tau_{RAW}(\nu) - w\, \tau_{H2O}(\nu) - \tau_{offset}$$

Finding the degradation factor for an observation corrupted by water vapor dispersion

To find out how sensitive our estimator will be to the brightness of a planet, **a(ν),** after making the data "waterless", we perform the following analysis.

Suppose that the original phase function was due to a true planet/star contrast ratio **a** which is constant over frequency. Then we would expect the measured phase to be:

$$\phi_{raw} = a\, R(\nu\tau_{sep}) + \phi_{offset} + \phi_{H2O} + \phi_{noise}$$

If there were no offset phase or water dispersion, then we could ignore those terms (as if the phase could be observed perfectly!) and the SNR^2 of an observation would be proportional to:

$$\text{Strength} = \int \mathbf{d\nu\, (W(\nu)\, \phi_{RAW}(\nu))^2}$$ (note we are now using W, not W')

Finding the degradation factor for an observation corrupted by water vapor dispersion

Now we do the same thing using the “waterless delay” which we first convert to an equivalent phase $\phi_{WL}(\nu) = 2\pi\nu\tau_{WL}(\nu)$

Then we expect an SNR^2 proportional to:

$$\text{Strength}_{WL} = \int d\nu \, (W(\nu)\, \phi_{WL}(\nu))^2$$

The degradation in the SNR is therefore given by:

D = $(\text{Strength}_{WL} / \text{Strength}_{RAW})^{1/2}$

Note that **a** will cancel out of this computation – this solely has to do with the resolvability aspect of the phase function.

D must be computed on a case-by-case basis. Some examples will now be presented.

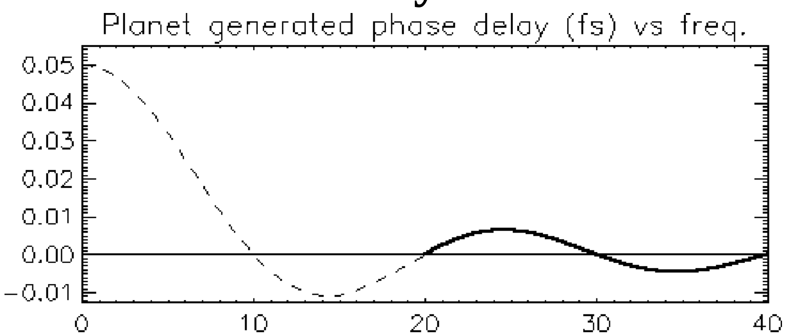
Examples of phase functions from planets, making those functions “waterless”, and finding the SNR degradation factors **D**

Assume detection in N band, from 20 to 40 THz, using constant weighting

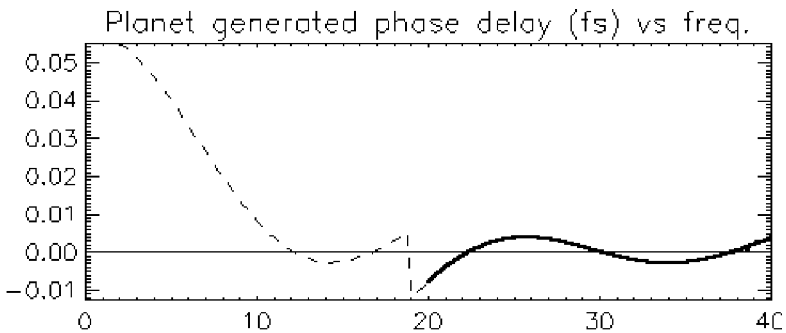
Well resolved observation:

τ_{sep} = 50 fs

Resolvability: 1.000 to 2.000



Underlying
Delay function



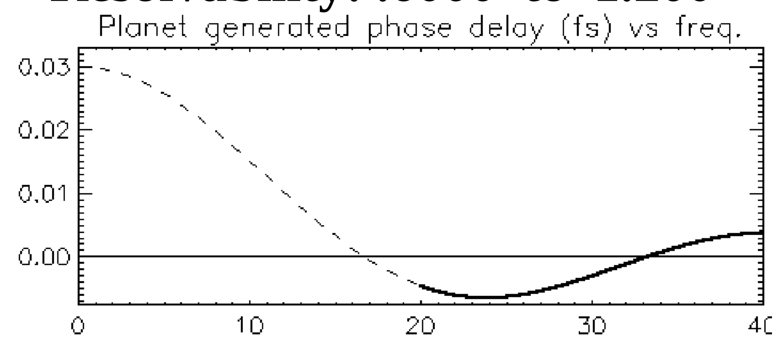
“Waterless”
Delay function

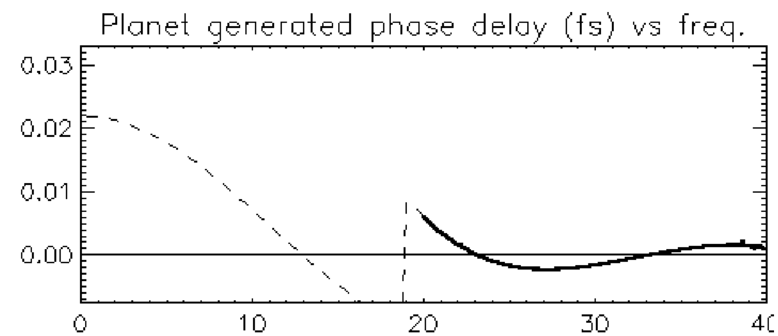
Degradation in SNR, D = .665

Resolved observation:

τ_{sep} = 30 fs

Resolvability: .6000 to 1.200





Degradation in SNR, D = .422

More examples

Barely resolved observation:

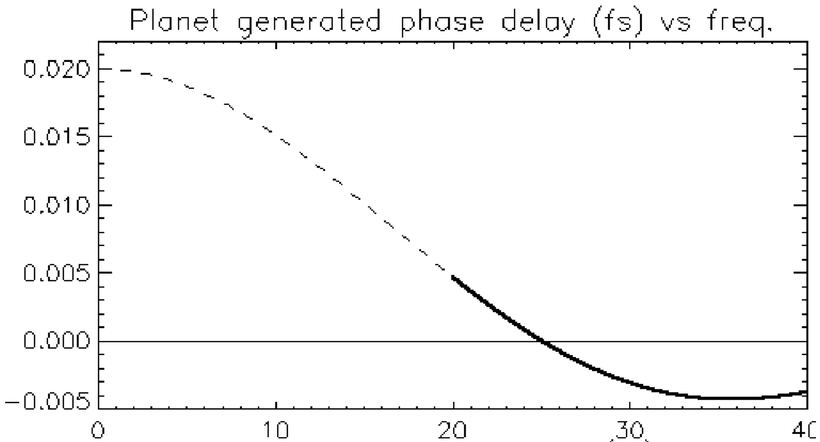
τ_{sep} = 20 fs

Resolvability: .4 to .8

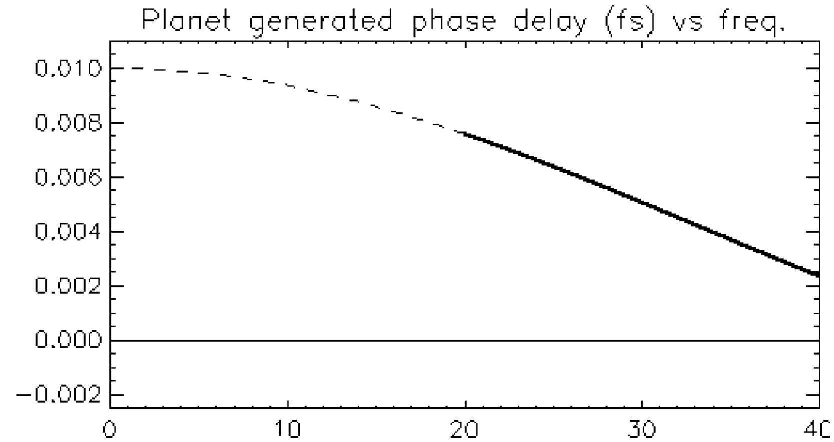
Under-resolved observation:

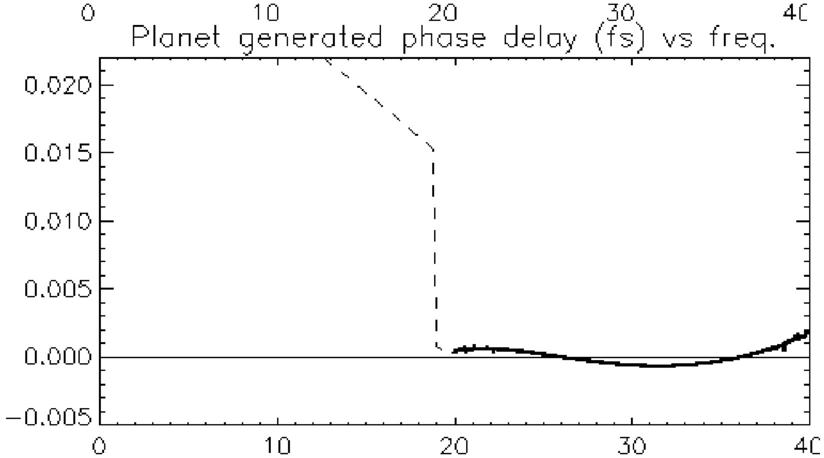
τ_{sep} = 10 fs

Resolvability: .2 to .4

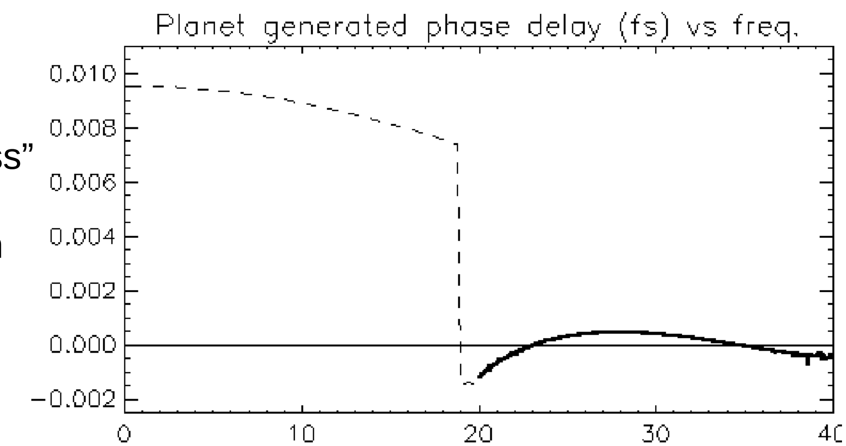


Underlying
Delay
function





"Waterless"
Delay
function



Degradation in SNR, D =.184

Degradation in SNR, D = .0752

Under-resolved observation using spectral features in source

$$\phi_{raw} = a(\nu)\, R(\nu) + \phi_{other}$$

In this case, the resolvability $r << 1$. That will cause the phase delay at all frequencies to be shifted by approximately the same amount (to the "center of light") so that the effect of the planet will be undetectable due to $R(\nu)$.

However if $a(\nu)$ contains sharp spectral features within the frequency range of observation, those will generally cause shifts in the phase delay which are clearly differentiated from a fixed delay, and cannot be ascribed to water-vapor dispersion.

Note: a planet spectrum which falls off at short wavelengths simply due to its temperature, is *not* sufficient for this purpose!!

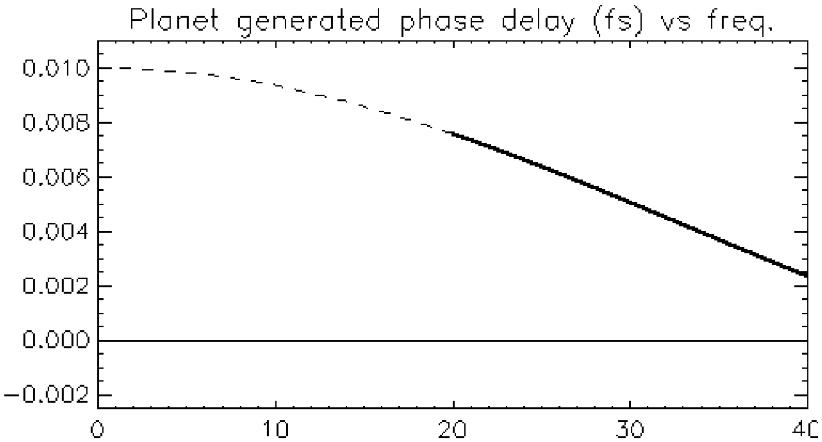
Example of enhancement of detectibility due to spectral features

Under-resolved observation:

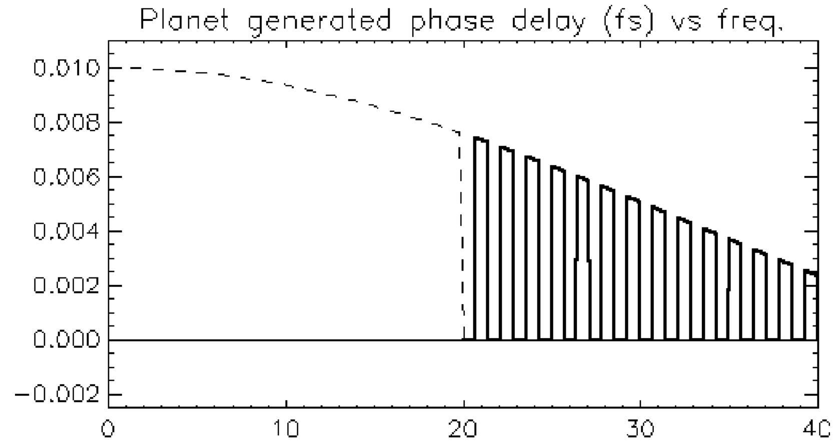
τ_{sep} = 10 fs

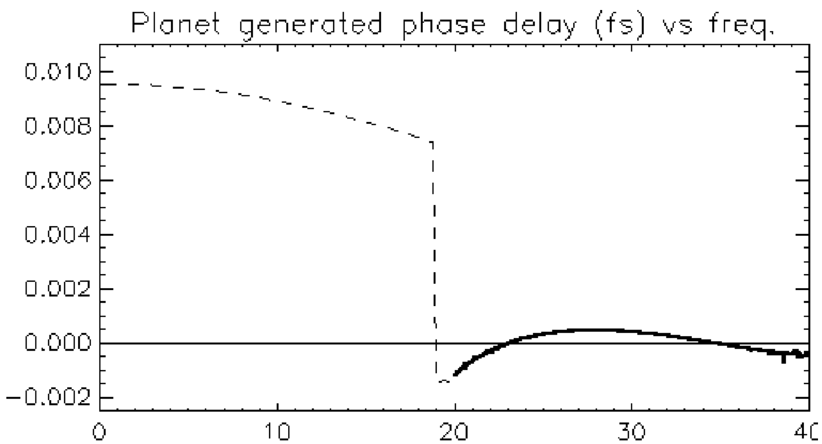
Resolvability: .2 to .4

Same planet, but with strong "absorption lines" across N band: will increase detectibility

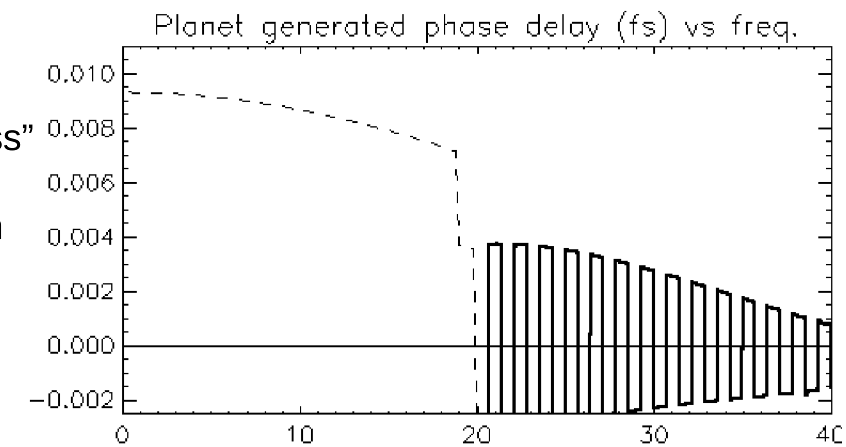


Underlying Delay function





"Waterless" Delay function



Degradation in SNR, D = .0752

Degradation in SNR, D = .714

The End

The VLT Array on the Paranal Mountain

ESO PR Photo 14a/00 (24 May 2000)