Interferometry

Approximate contents of the course:

- Why bother?
- Optics, mostly wave optics, necessary to understand that which is interfering
- Some (not much) quantum optics. What is actually interfering with what?
- Coherence and power: how do we add the electric fields from many independent sources
- Basic components of a realistic optical/infrared interferometer
- Delay lines, who needs them?
- Atmospheric constraints
- Differences between radio and optical interferometers
- Sizes and configurations of real oir and radio interferometers
- Detectors for interferometry
- Sensitivity calculations
- Fourier Transform techniques
- Image reconstruction techniques and problems
- Polarization effects
- Oddball interferometers e.g. intensity interferometers, Fourier Transform Spectroscopy

A more detailed list of topics is here:[interferometry topics]

Why Bother?

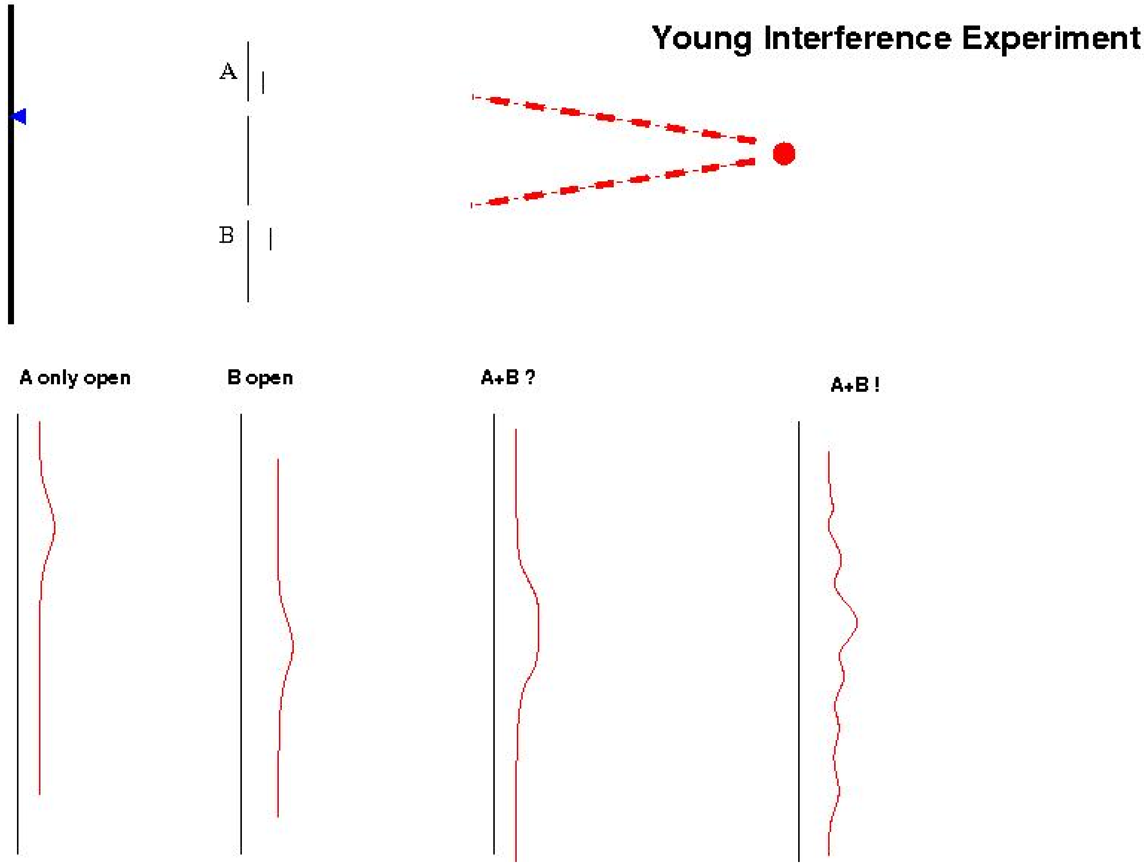
All of physics is interferometry!

All of optics is interferometry!

Interferometry is a relatively cheap way of achieving very high spatial (and sometimes spectral) resolution.

The resolution of a telescope is ~λ/D and λ is fixed by the physics you want to investigate, so higher resolution is obtained by increasing D. However the cost of constructing and instrument usually is approximately proportional to the weight or D^3 This gets very expensive very quickly. By throwing away most of the telescope aperture we can reduce the expense at the cost of sensitivity and interpretability. We will consider the questions of image reconstruction and interpretation in detail in later lectures.

Do we believe in interferometry?



How does this work?

$$E = E_A + E_B = E_o[\exp(i\phi_A) + \exp(i\phi_B)] =$$
$$E_o[\exp(2\pi i D_A / \lambda) + \exp(2\pi i D_B / \lambda)]$$

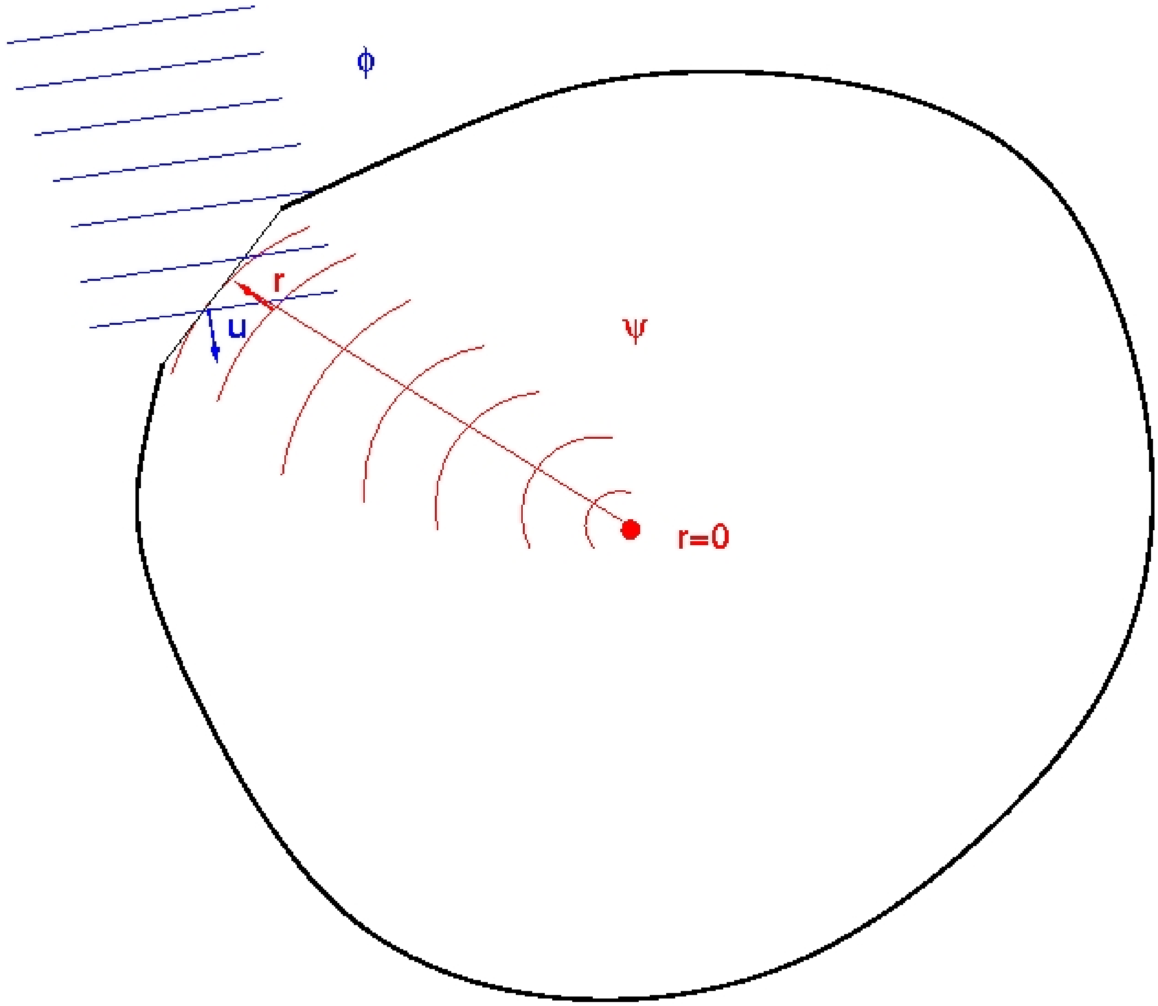
So much for the electric field. But the received power is proportional to the squared amplitude of the field:

$$P = P_o(1 + 1 + 2\cos((2\pi / \lambda)(D_A - D_B))$$

If we close off either ***A*** or ***B*** the wiggles disappear. This is of course obvious in classical electromagnitism but becomes a littly murky in quantum physics.

Since $D_A - D_B \propto L\sin(i)$, where ***L*** is the baseline and ***i*** the angle of incidence, it is easy to get a high resolution with a large L.

In order to understand how interferometers work in more complicated circumstances: extended sources, finite sized apertures, finite wavelength intervals… we have to learn a little bit about wave optics and this is easiest to do by considering Huygens construction of wave
propagation.



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Now it is nice to make drawings with wiggly lines, but to do reasonable calculations we need support of some mathematics, based on a Green's function approach to Maxwell's equations:

We want to find an expression for the electromagnetic at some point (e.g. the origin) expressed in terms of its value over some near or far surface, after propagation to our point of interest. This is essentially a *Green's function* problem for Maxwell's equation which we will write in the form (for waves of fixed frequency):

$$\begin{aligned} \nabla^2\phi &= \frac{\partial^2\phi}{\partial t^2} = -k^2\phi \\ k &= n\omega/c \end{aligned}$$

We will make the wild guess that the function $\psi(r) = \exp(ikr)/r$ has interesting properties. This is motivated by the fact that it is almost a solution for Maxwell's equation in vacuum:

$$\nabla^2\psi \quad + \quad k^2\psi \quad = \quad 4\pi\delta^3(r)$$

And now a lot of manipulations of space/surface integrals with Green's theorem:

$$\begin{aligned} 4\pi\phi(0) &= \int_V \phi(\nabla^2\psi + k^2\psi)dV \\ &= \int_v (k^2\psi\phi - \nabla\phi \bullet \nabla\psi)\ dV \quad + \quad \int_S \phi\ \nabla\psi \bullet \vec{n}\ dS \\ &= \int_V (k^2\phi\psi \quad + \quad \nabla^2\phi\ \psi)dV \quad + \quad \int_S (\phi\nabla\psi - \psi\nabla\phi) \bullet \vec{n}\ dS \\ &= \mathbf{0} \quad + \quad \int_S (\phi\nabla\psi - \psi\nabla\phi) \bullet \vec{n}\ dS \end{aligned}$$

where $\vec{n}$ is the vector normal to a surface element surrounding the origen.

This now has the form of a somewhat opaque surface integral. If we assume that $kr >> 1$ and that the gradients of the incoming wave ϕ are due primarily to the waves k then the gradients of ϕ and ψ are equal to the functions themselves times $ik\vec{u}$ where $\vec{u}$ is a unit vector in the direction of propagation.

So finally:

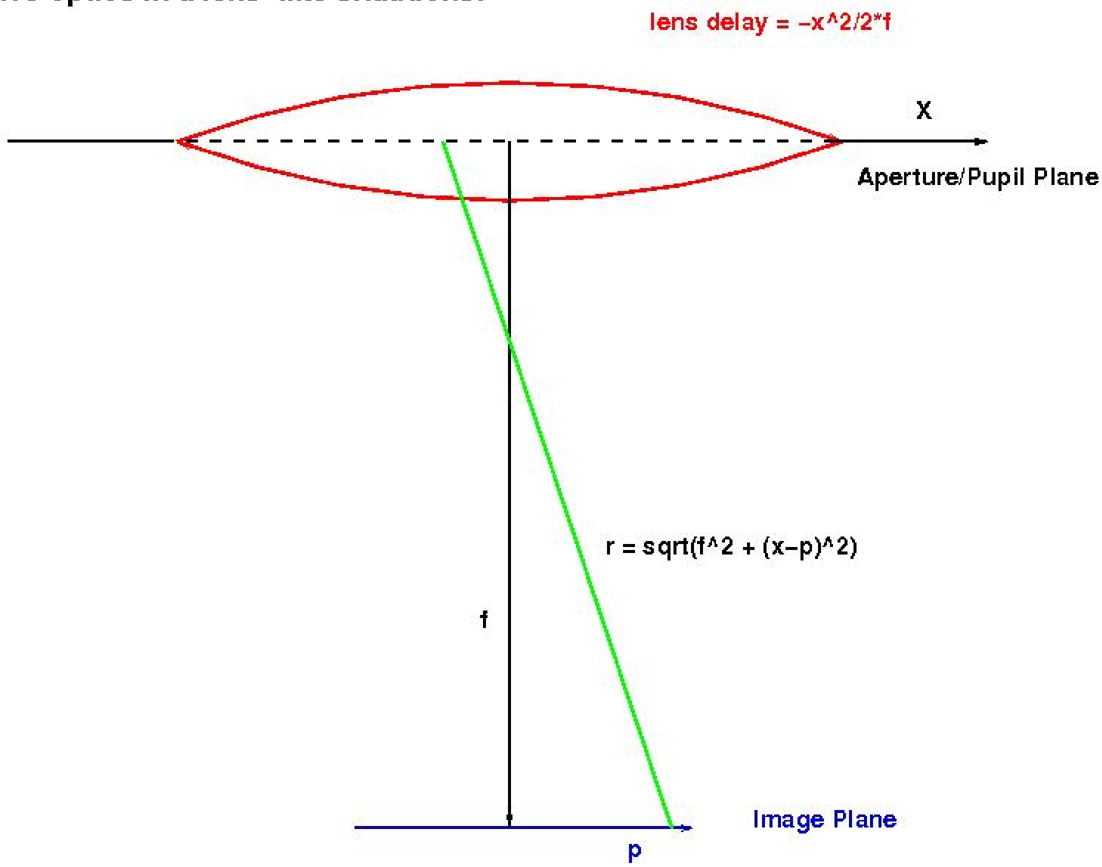
$$\phi(r = 0) \quad = \quad \frac{i}{4\pi\lambda}\int_S \phi(S)\exp(ikr)/r\ (\cos(n,r) - \cos(n,u))\ dS$$

where the terms in the last integral represent the field on the surface, a *propagator* from the surface to the origin, and some terms representing the angles between the surface and the incoming rays.

Homework: Demonstrate that $\nabla^2\psi + \boldsymbol{k}^2\psi = 4\pi\delta(0)$ for the above definition of ψ.

Immediate application: how does a lens work?

Wave optics in a lens-like situations:



Again the
mathematics:

$$
\begin{aligned}
E(\vec{p}) &= \frac{ik}{r}\int E(\vec{x})e^{ikr} \\
r^2 &= f^2 + (x-p)^2 \\
r = \sqrt{f^2+(x-p)^2} &= f\sqrt{1+\frac{(x-p)^2}{f^2}} \\
r \simeq f\left(1+\frac{(x-p)^2}{2f^2}\right. &= f + \frac{(x-p)^2}{2f} \\
E(\vec{p}) &\simeq \frac{ik}{r}\int E(\vec{x})e^{ikf}\exp\left(\frac{ik(x^2-2xp+p^2)}{2f}\right) \\
&\simeq \frac{ik}{r}\exp\left(ikf+\frac{ikp^2}{2f}\right)\int E(x)exp\left(ik(\frac{x^2}{2f}-\frac{xp}{f})\right)d^2x
\end{aligned}
$$

The lens is designed to add an extra **optical path delay (OPD)** of $\Delta Z = \frac{-x^2}{2f}$ with the result that:

$$E(\vec{p}) \quad \propto \quad \frac{ik}{r}\int E(\vec{x})\exp(\frac{ikpx}{f})d^2x$$

which is simply the Fourier transform of $E(x)$. So a lens acts as an *analog* fourier transformer of the electric field.

Some simple examples: consider a square aperture of side D. Can you show that the electric field in the image plane is: $(i/2\pi\lambda f)D^2 * 4\mathrm{sinc}(D\alpha/4\pi\lambda)\mathrm{sinc}(D\beta/4\pi\lambda)$

Where sinc(x) = sin(x)/x and α and β are the two coordinates in the *p* plane, measured as angles on the sky. The **brightness** detected in the image plane is the square of this. You see that the brightness is proportional to D^4. But the energy going through the hole is proportional to D^2. Can you explain this?

Can you calculate the brightness pattern for the light going through a circular aperture of radius *r*?