Noise in Optical Interferometry

While we are still on noise calculations let's make some similar calculations for optical/IR interferometry, although we haven't dealt with the details of the instruments yet. The first is that we are now in the regime where $h\nu > kT$ where I haven't specified exactly what I mean by *T*, but you can take it to be the brightness temperature of the object+background that we are looking at. So <*n*> <<1 and the standard deviation in the number of photons is SQRT(<*n*>).

Note that these statements are definitely true for optical and near infrared observations, but only marginally true for midIr observations (say 10μ wavelength and T~300 K).

Less fundamental but equally important for groundbased interferometry: The atmosphere introduces rapid phase fluctuations so signals cannot be coherently added indefinitely.

The maximum integration time is about 10 msec for visual/near IR observations and scales with wavelength approximately as $\lambda^{6/5}$

Thermal formulas for counting photons etc are still relevant, but more complicated to use because they no longer are linear in T.

The final signal is still the number of coherent photons from the source, while the noise is the quadratic sum of uncertainties from:

- Photon noise from the source (usually not important)
- Photon noise from thermal backgrounds of sky or telescope (important for mid infrared)

- Detector readout noise (typically dominant at shorter wavelengths.

I'll give some typical calculations of these quantities:

Background Noise: As in the radio case, lets assume that the thermal background looks like a black body with an emissivity ε.

We rewrite the BB law: $B = (2kT/\lambda^2)(h\nu/kT)/(\exp(h\nu/kT)-1)$

In terms of *photons*/s/cm^2/ster/Hz by dividing by the photon energy: $(2/\lambda^2)/(\exp(h\nu/kT)-1)$. Now consider the photon number in time *t*, in bandwidth Δv from a telescope with

diameter *D* solid angle $(\lambda/D)^2$: $2\varepsilon\Delta\nu t/(\exp(h\nu/kT)-1)$. Note surprisingly that it doesn't depend on the telescope size or the wavelength. Orders of magnitude: in the midIr, v~3*10^13Hz, so

Δv might be 10^13 Hz, t might be a few tenths of a second before atmospheric decorrelation, ε depends on the exact wavelength, but also contains significant contributions from the telescope mirrors, perhaps even 50% for a complicated interferometer.

bands

So the number of photons/s/telescope is fairly large except for the exponential factor 1/(exp(hc/λkT)-1) where hc/k = 1.439 cm K.

Homework: at which wavelength is the number of sky photons per second of order unity?

If the background counts were exactly constant, they would be no problem, but of course in any particular interveral they show a standard deviation of order SQRT(<n>).

Readout Noise is caused by thermal fluctuations in the electronics that are used to convert the –very small—electric charge that has been read into an amplifier, into a much larger signal. This uncertainty is stated as the r.m.s. uncertainty in the number of electrons created from a photoelectric device per readout per pixel.

This noise varies with the device, its temperature, and the speed at which the device is read out, varying typically as 1./sqrt(t_readout).

Typical values are ~5 electrons (visible CCDs), 30 electrons (near IR CdHgTe devices), 600 e- (midIR SiAs detector).

So the thermal noise and readout noise do not (to first order) depend on the size of the telescopes, but the **signal** does, so the S/N does.

For signal strengths expressed in *Janskys* this is fairly obvious:

Home work: how many photons/s come from a 1 Jy source at

10μ wavelength with a 10% bandwidth into a 10 m telescope?

Of course expressed as the signal from a blackbody source the same scaling with diameter applies;

Homework: calculate photons/s for the same telescope from a 300 K opticaly thick target with an angular size of 10 millarcsec.

Now a particularly nasty aspect of noise in optical interferometers: you cannot amplify the signals from a telescope without destroying the phase information. So if you want to combine the signals from two telescopes, the S/N is sqrt(2) better than 1 telescope (actually depends a bit on the details), but if you have more than 2 telescopes, you have to split up the light with beam-splitting mirrors and combine it in bits and pieces. Suppose you have *N* telescopes and you try to

detect all (*N*(N-1))/2* visibiities. Then the light available at each beam combiner is ~2/*N* of the light from each telescope, This leads to a *decrease* in S/N for each visibility of 2/*N* (readout noise limited)

or SQRT(2/*N*) (background limited). If we can combine all the visibilities, say to estimate the flux of a point source, the S/N is increased once again by SQRT(*N*N*/2), so the S/N relative to a single interferometer pair (or a single telescope, for that matter) is proportional to SQRT(*N*) (background limited) or $\boldsymbol{N}^0$ (RO noise).

There are some tricks to improve this latter result – to be discussed later—but this line of reasoning discourages building large arrays of optical telescopes, and most systems are limited to 4-6 telescopes.

Some myths and facts about coherent and incoherent noise.

With a nice radio interferometer under good weather conditions, the atmospheric phase distortion is small during the time that it takes to calibrate it, so individual complex visibilities are *coherent*. This means that if you measure a large number of them in a short time, you can add them as complex numbers, and the S/N of the result is that of the individual measurements but improved by SQRT(*N_DIT).*

Simple complex example: Suppose the signal is $c+is=A\exp(i\alpha)$ and the noise in the real and complex measurements is $\sigma(g_c + ig_s)$, where the g's are the usual gaussian variables. If you **add** N measurements the signal becomes $Nc+iNs$ while the noise becomes $\sqrt{N}\sigma(g'_c + ig'_s)$, so the S/N of the result is ~A/σ

In the optical domain, and to some extent in the VLBI domain, the sky changes the phases of the complex visibiities in a period shorter than one can calibrate (unless a reference source can be used). Then it is not possible to determine the full complex visibilities but only their

amplitudes. In this case we cannot just add up the measured complex visibilities because they decorrelate. If we extend the above example, the noise calculation is unchanged but each realization of the signal is multiplied by exp(iϕ) where the ϕ are random numbers.

One can show that < exp(iϕ)> = $\exp(-<\phi^2>/2)$.

Homework: show that this is true.

For large rms values of ϕ the average values of the signals *c* and *s* disappear. To make the algebra in the following sections simpler, we assume that ϕ_{rms} is large (in radians). The *rotated* vaues of the real and complex parts of the signal are now random variables with

rms values of A/sqrt(2) (where does the sqrt(2) come from ?) If there are a lot of them they can be considered to have a gaussian distribution. So suppose we decide to average them together.

We write each complex measurement as $M_i = (A/\sqrt{2})(\gamma_1 + i\gamma_2) + \sigma(g_1 + ig_2)$ where the g's and γ's are the usual normalized gaussian variables. Clearly <M>~0, so using the average of the complex values to estimate *A* doesn't work. What if we, foolishly, take the *square* of the average, which is at least positive.

Let $T=\Sigma M$. With the usual manipulations, $<T>=0$.

$<T^2>=N(A^2+2\sigma^2), Var(T^2)=N^2((3/4)A^4+3\sigma^4+4\sigma^2A^2$ give or take a factor of 2. We see that we could use this method to estimate *A* if we know σ, ($A \sim \sqrt{<T^2>/N-\sigma^2}$), but the uncertainty in this estimate does not decrease with *N*!: standard deviation(T^2/N)~$A^2+\sigma^2$.

Bad idea. A better idea is to take the absolute value of each measurement, squared, and average these:

$<|M|^2>=A^2+2\sigma^2; Var(|M|^2)=(3\sigma^4+A^2\sigma^2)/N$

A couple of things to notice—

The S/N now goes up as sqrt(*N*).

If (σ<A) the S/N is of order $\sqrt{N}A/\sigma$ as expected

If (σ>A) the S/N is of order $\sqrt{N}(A/\sigma)^2$

Master's research project (small):

Work out a theory to estimate *A* from *M* in the *partially* coherent case where ϕ_{rms} ~1.