A Little More Fourier Transforms

Fast Fourier Transforms: Performing a FT by multiplication by a bunch of sines and cosines is called a *Direct Fourier Transform* (DFT). If you count the number of multiplications it takes to finish the job, it takes (of order) N^2 complex multiplications. For large numbers of points this can grow relatively large. In the 1800s Gauss conceived a numerical shortcut now termed a Fast Fourier Transform (FFT) and named after Cooley and Tukey who rediscovered it in 1964. The basic idea is that an FT can be factored into a set of smaller FTs that take fewer operations (see http://en.wikipedia.org/wiki/Cooley-Tukey_FFT_algorithm) . The number of operations is then ~2N(sum of prime factors of N). If $N = 2^n$, which is a common case, then $\text{NOP} \approx 2N \log_2 N$. This can lead to factors of 1000 improvement, and it is almost universally used in numerical computation.

The big loser is that the algorithm requires that the input data must be uniformly spaced. This may or may not be the case. If it isn't, and you still want to use a FFT, it is necessary to *interpolate* the data onto a uniform grid. You can do this by a straightforward interpolation, but the process invariably destroys information in an unpredictable way. The preferred process is 2-step. First we convolve the data with a smooth function, destroying high frequency data in a controlled way, then multiplying by a Shah function (resampling/reduplication in the FT space) and then a FFT. The main error is that a pragmatic smoothing function leaves a little high-frequency information in the FT plane, which is confused with information from the next duplicate

piece of FT space. This confusion is called *aliasing*. It can be suppressed by using a fancier smoothing function, or by increasing the density of sampling points (pushes the FT tiles farther apart), but both fixes cost computing time

A more or less complete radio interferometer

Let's mix and match components and build a radio interferometer. We want to measure the E field at two separated pieces of ground, and then correlated them.

So first the collection, with an antenna. This may be *fixed* conductor (e.g. a dipole) or a steerable, often parabolic *dish* (collector). These have the advantage of high sensitivity in a specific direction, but are more expensive. Often high frequency systems have steerable dishes, and low frequency systems have fixed dipoles.

In very large modern telescopes like *LOFAR*, the individual antennas are simple dipoles, but groups of the dipoles are combined together as *phased arrays* to limit the field of view and lower the data rate of transmission. Phased arrays are complicated and expensive but offer great advantages in flexibility and speed over physically steered arrays.

Then you want to bring the signals to a common point. Mirrors etc are a pain, so this is usually done by bringing the signals via wires or coax cables or wave guides. At very high frequencies these do not work very well, so it is common to reduce the signal frequency by *heterodyne* mixing to a lower frequency. If we want the final correlation to be phase stable, it is necessary for the mixing signal

(*local oscillator*) to be phase stable. This is done in a geographically local system (not VLBI) by generating a single, relatively low frequency, signal at a central point, then transmitting it to the antennas, then creating a high frequency signal with a phase stable multiplier.

There are several important technical limits on signal transmission:

1. Little loss of signal or addition of noise.

2. Bandwidth: needed for good S/N; ~1 GHz for electronic transmission, say 10 GHz for optical fibers, 1 PHz for mirrors

3. Phase stability w.r.t. physical or atmospheric changes.

Delay Lines:

After transmission, the signals from the telescopes are brought to a central point, amplified(!), and prepared for replication and correlation. A crucial step at this point is the insertion of delay lines. These bring the difference in path length from the various telescopes, including the geometric delay on the sky, to *almost* zero.

Why? The typical delay,OPD, is of order *B*, the baseline, which is many wavelengths. The delay also varies rapidly with time, (how rapidly?), and "slowly" with position. From the FT correlation theorem that we proved earlier, the strength of the correlation is proportional to FT(Power Spectrum; OPD), which begins to drop when $\boldsymbol{OPD} * \Delta\nu > 1$, where $\Delta\nu$ is the *bandwidth.*

We could avoid any decorrelation by choosing a very small bandwidth, but this has a bad effect on our signal/noise result. So we reduce this problem by inserting a *delay line* for each telescope that cancels the *OPD,* **at one specific point on the sky,** known at the

tracking center. This delay correction must track the change of delay with time. The *gradient* of delay with position, the notorious UV coordinates around the tracking center, is not changed by this operation, so the whole image synthesis business still works. If the field of view is large, the OPD at the edges can violate the above criterion, leading to a loss of correlation. This is technically known as *delay attenuation* or *delay beaming.* If this is a problem you have to decrease the bandwidth, for example by breaking the total band up into subbands.

Radio delay lines were first constructed with switchable pieces of coax cables, but this is messy, inflexible, and the switches add difficult to calibrate changes in transmission. Modern delay lines are digital. The signal is gain stabilized, digitized or *sampled* (see below) with an ADC (*analog digital converter*), and introduced into a large bank of shift registers. The sampling rate, and time accuracy of the shifting are driven by the required bandwidth. In current radio usage this often lines in the range of a few MHz to a few GHz.

Replication and Correlation:

We now figure out how many telescope pairs we wish to correlate, and break up the incoming signals into enough copies to do this. Since this is all digital processing it is fairly straightforward.

Then we *correlate* the signals. In theory is consists of multiplication and averaging. Because full fledged digital multipliers are slow and complicated, a simplifying approximation is used: the signals are brought to a standard level relative to the noise, and 1-bit sampled, i.e. either positive or negative. The samples from two telescopes are "multiplied" by a very simple circuit called an **exclusive OR (XOR)**

circuit (1*1=1,0*1=0,1*0=0;0*0=1), and then accumlated. This can easily be done at multi GHz rates.

HOMEWORK: What is the loss in signal/noise ratio that occurs with 1-bit multiplication versus true multiplication? Assuming that the instantaneous S/N of the incoming signals is very small.

In practice it turns out to be cheap enough to use slightly more complicated multipliers (1.5 or 2-bit) that reduce the noise somewhat relative to a 1-bit multiplier.

Complex correlation

A direct multiplication of the incoming signals yields something proportional to the **cosine** of the OPD. If we want to do a true FT to get an image, we also need the **sine** component. This is done by setting up a 2nd correlator system with a $\lambda/4$ delay on one telescope of each pair. Note that this only works if the bandwidth is relatively limited, otherwise $\lambda/4$ is not well defined.

Note also that for the imaging FT to work right you need to get the *phases* of the correlations right, as well as their amplitudes. This means that you have to keep phase/time stability of the entire receiver, transmission, delay, and correlator systems accurate to $\lambda/4$ or better.

Spectral Line Correlation:

In many astrophysical cases we are also interested in the spectrum of the signal, often as a function of position. Can we arrange our interferometer to deliver spectra? This may also be necessary to decrease the delay limitations on field-of-view. In the optical/ir this is

done, rather crudely, by inserting prism/grating hardware before the correlator. In digital radio systems this can be done by inserting *variable (multiple)* delay lines, so that the correlation is measured at a range of delays. The correlation as a function of frequency can be recovered by an addition Fourier Transform.


A Note on Calibration:

Many of the components I have described affect the amplitude and phases of the signals delivered to the correlator. Generally these distortions cannot be calculated directly from the component designs, and most vary with time. If they are not known, however, they prevent accurate image construction. Usually we only try to specify in the design that they change "slowly" with time and we measure the *end-to-end* distortions of the atmospheric, reception and transmission train by observing **calibration** sources of known strength and position. We need to observe them more frequently than the rate of *change* of the component characterization.

A Second Note on Calibration:

It often turns out that this is not possible for physical reasons, e.g. we cannot switch the antennas from source to source that quickly. A partial solution is *self-calibration* in which we identify some *redundancies* in the signal system and use these to constrain the instrumental behavior. I will discuss these more completely in the sections on data reduction.
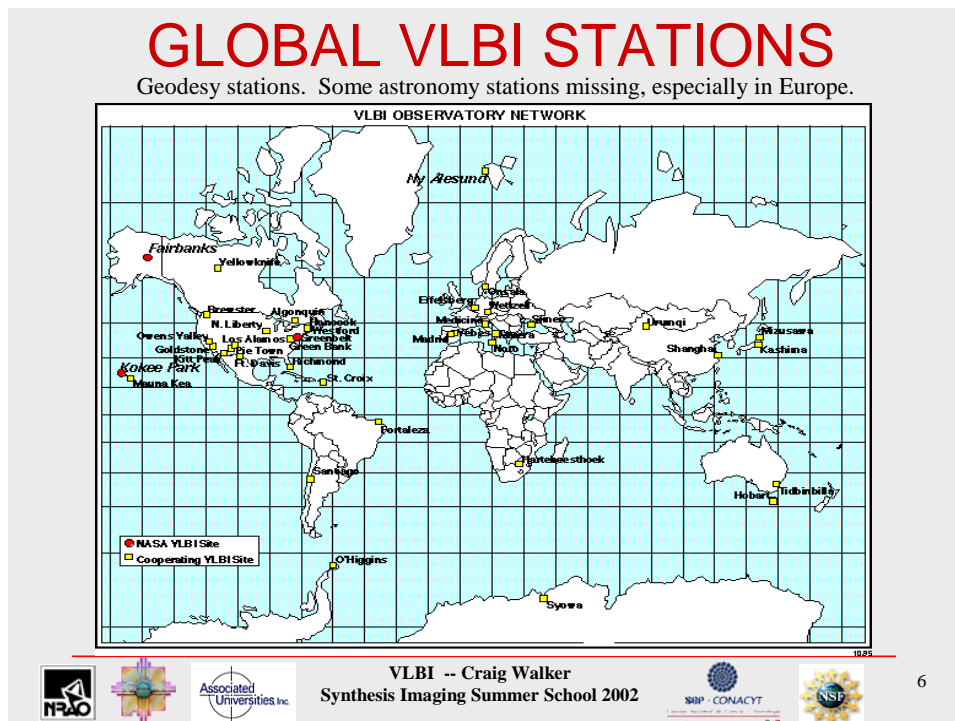
Pretty Pictures:

© Spektrum der Wissenschaft/Emde-Grafik
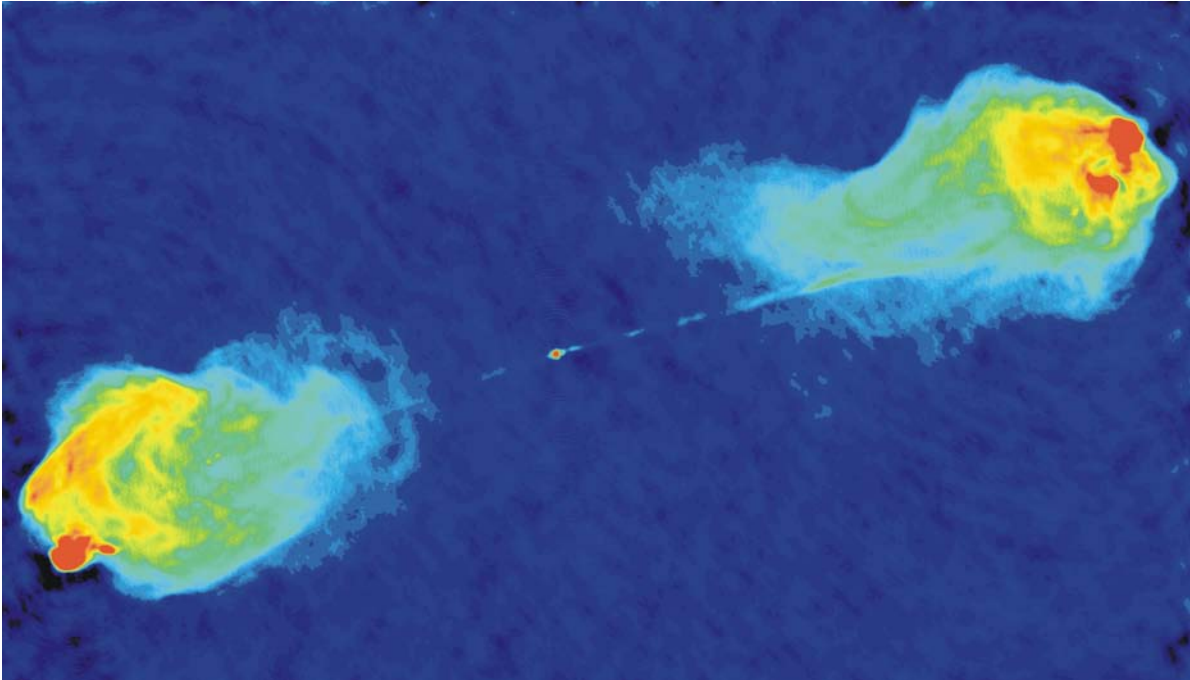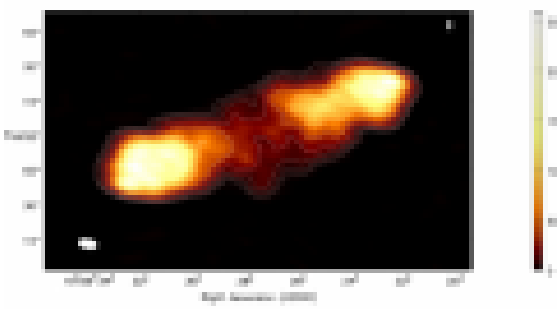
VERY LONG BASELINE INTERFEROMETRY (VLBI)

see also:

This is a magical/mysterious extension of radio interferometry where the IF (intermediate frequency) signals from the various telescopes are *not* brought together directly, but recorded on magnetic tape and brought together days-to-weeks later for electronic correlation. The advantage is extreme high resolution:for example if we observe at 3mm on a baseline of 10,000 km the resolution is 3.10^{-10} radians (300 picoradians, or 6 microarcsec). The disadvantages are high operational complexity and a limited number of targets because of low surface brightness sensitivity.
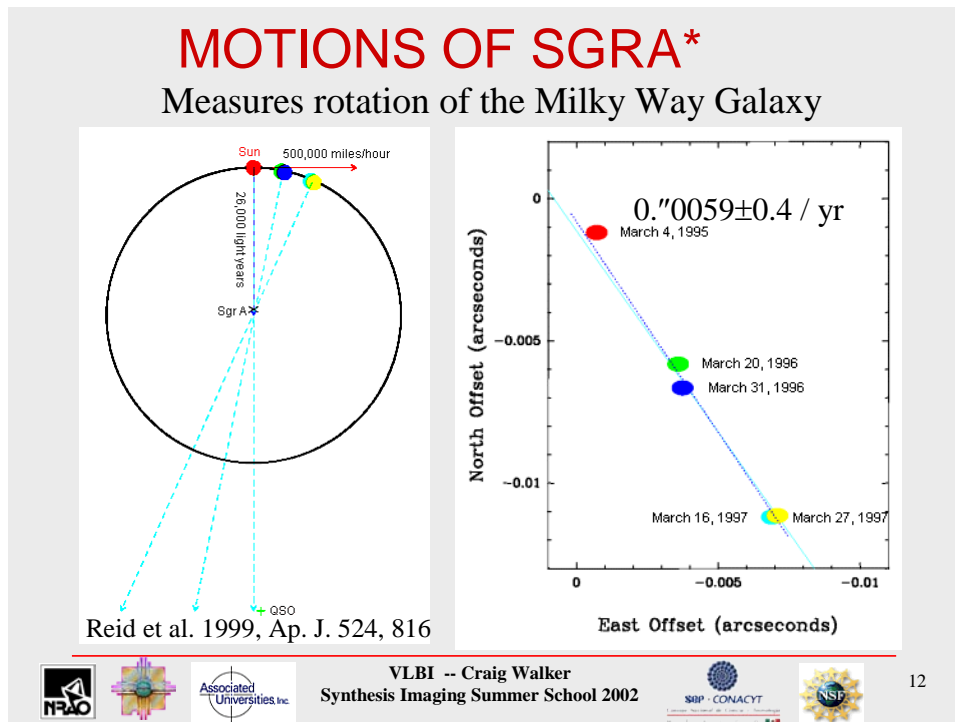
VLBI stations:

Pretty VLBI pictures:

3C120 superluminal motion

m87movie

MOTIONS OF SGRA*
Measures rotation of the Milky Way Galaxy

$0.''0059 \pm 0.4$ / yr

Reid et al. 1999, Ap. J. 524, 816

VLBI -- Craig Walker
Synthesis Imaging Summer School 2002

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Special issues of VLBI observing:

o Telescopes are often not specifically built for VLBI usage

o To get delay right to (say) 100 nsec is not easy:

There is no synchronisation signal sent to all telescopes so we use very accurate clocks (hydrogen masers) at each site and synchronize one to the other using e.g. GPS. The maser signals are written on the tape along with the data. The true delay is found empirically by trying to find fringes on calibration sources and tracking the delay and delay rate, and interpolating between calibrators.

Raw Correlator Output Phases
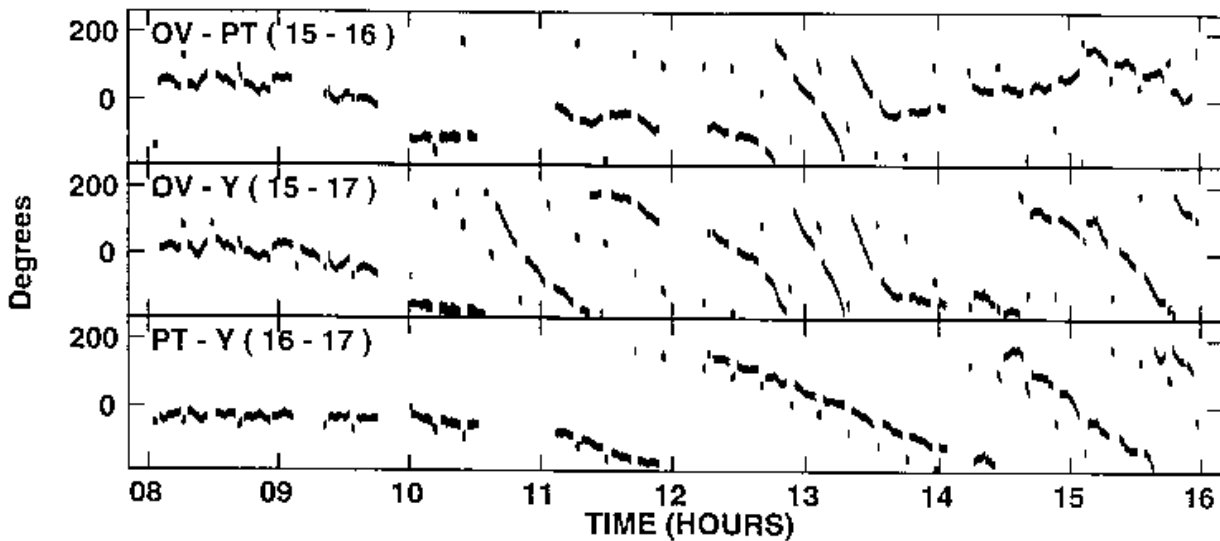
Before Calibration
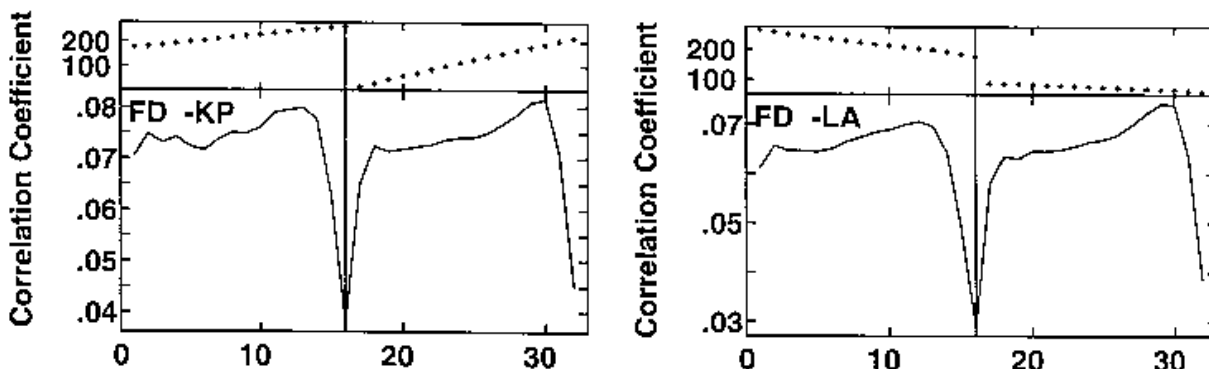
o Delay depends on distance and orientation between stations, to mm accuracy. If one has a reliable set of calibrators, this information can be used to measure the positions of the stations including continental drift, earth rotation etc.

o VLTI determination of telescope positions and quasar positions help define the **International Celestial Reference Frame** and the **International Terrestrial Reference Frame.**

See also: IERS home page, US Naval Observatory ICRS page