

Examination Cosmology 2006

Thursday 12 January, 10:00-13:00

Room F320 UvA

Vincent Icke

*Every sub-question (a, b, c...) has the same weight in the final grading. I recommend answering those questions first which you find easiest. You are allowed to use only your own handwritten notes, as well as the Basic Material for Cosmology which I placed on my web site. You may use a pocket calculator, provided that it isn't programmed as a crib sheet. On each sheet, write your **name and student ID number**, write your **address** on the first sheet, and number the sheets. In your answers, write down everything you deem applicable, including intermediate steps. You are allowed to hand in rough work, provided that it is marked as such. In each answer, give your argumentation in sufficient detail! Be as exact and quantitative as possible (i.e. use equations). Lack of clarity will invariably be held against you.*

$$G = 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$$

$$c = 3.00 \times 10^8 \text{ m s}^{-1}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$\hbar = 1.05 \times 10^{-34} \text{ J s}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

(1). _____

The standard description of the global Universe is the Friedmann-Robertson-Walker (FRW) family of models. These are solutions of General Relativity Theory (GRT).

- a. Why does the velocity field in FRW models obey the rule $\vec{v} = H\vec{r}$? Why does this allow us to use a Newtonian equation of motion for the evolution of the mean distance R between objects as a function of the mean volume density ρ of matter?
- b. Why then is the use of GRT unavoidable?
- c. Write down the Friedmann equations and show how, in the special case of a 'dust' universe, only one equation remains. How can the various terms (in principle) be determined from observations?
- d. Write down the first Friedmann equation for a fully relativistic equation of state. Show that it follows from this and (c) that the early Universe must have been relativistic, and give the solution of the scale factor $a(t)$ for very early times.

(2). _____

The cosmic microwave background radiation (CMBR) is possibly the most important observable quantity in our Universe: the 'glow of the Big Bang', observable as an almost exact blackbody radiator with a temperature of 2.735 K. The deviations of the observed intensity from a perfectly spherically symmetric distribution are usually analyzed by decomposing them as a sum of spherical harmonics Y_{lm} .

- a. Which are the main three effects that can give rise to a temperature fluctuation $\Delta T/T$? Which one is probably the dominant one in our Universe?

- b. Give arguments why the ‘azimuthal quantum number’ m is irrelevant.
- c. Consider Fig.1, showing the normalized amplitude of the CMBR fluctuations as a function of the ‘multipole quantum number’ l . Describe qualitatively how the wiggly form of the curve is related to the behaviour of small-scale linear fluctuations in the cosmic mass density.
- d. Describe qualitatively what happens to the amplitude $A(D, t)$ of an infinitesimal fluctuation with a given linear size D as a function of time t , starting at a time that is only a little later than $t = 0$.

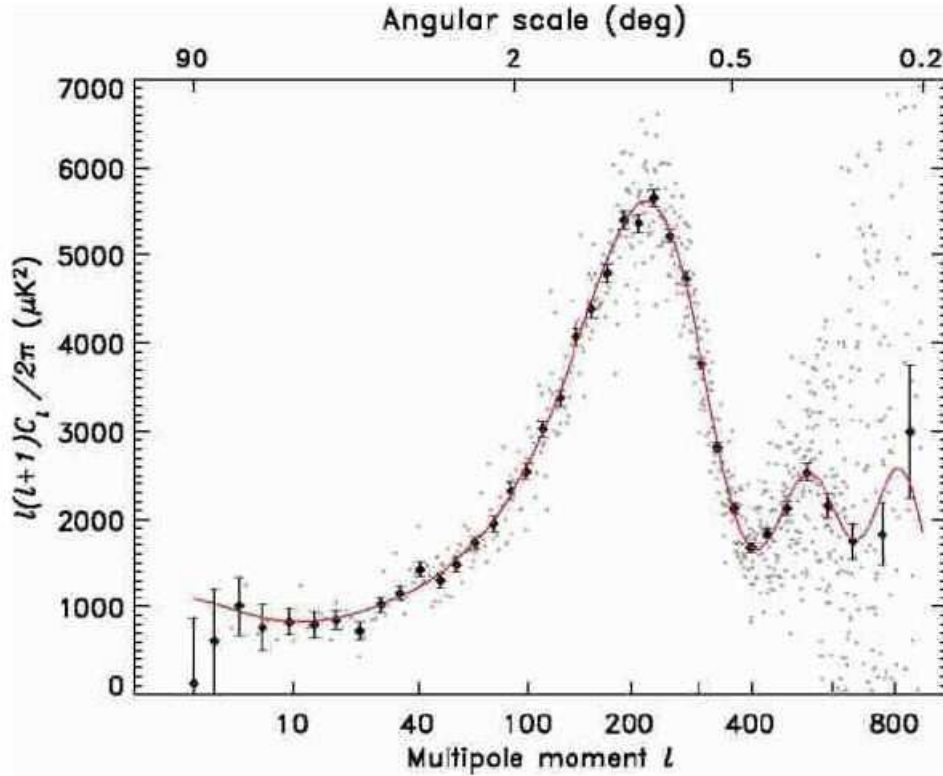


Fig.1. Observed amplitude (abscissa) of the CMBR fluctuations as a function of the l ‘quantum number’ of the spherical harmonic Y_{lm} (ordinate).

- e. Which are the main three effects that could influence the evolution of A as a function of time?
- f. Which of these three effects is probably not present in the dark matter component of the Universe?